CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE MATHEMATICS AND BACHELOR OF SCIENCE IN ACTURIAL SCIENCE

MATH 125: DISCRETE MATHEMATICS

STREAMS: "AS ABOVE" **TIME: 2 HOURS**

DAY/DATE: FRIDAY 07/12/2018 **11.30 A.M. – 1.30 P.M.**

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** Questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE (30 MARKS)

a) Use mathematical induction to prove that $n^2 \geq 2n+1$ for $n \geq 3$ (5 marks)

b) Translate the logical equivalence $(T \wedge T) \vee \neg F = T$ into an identity in Boolean algebra (3 marks)

c) Describe a Lexographic ordering in a set $A_1 \times A_2$ where A_1 and A_2 are linearly ordered sets. Use this order to explain why (3,4) $\prec (4,1)$ (3 marks)

d) Verify whether or not the relation represented by the matrix below is either reflexive or symmetric. (4marks)

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

e) Solve the linear congruence equation $4x \equiv 6 \pmod{10}$ (4 marks)

f) Find the product of the polynomials $f(x) = 7x^3 - 4x^2 + 3x - 11$ and $g(x) = 15x^3 + 3x^2 - x - 14$ over Z_5 (4 marks)

g) Let R be a relation defined on the set $A = \{0,1,2,3\}$ containing the ordered pairs (0,1), (1,1),(1,2),(2,0),(2,2) and (3,0). Determine

i. The reflexive closure of R

ii. The symmetric closure of R (4 marks)

h) Determine the validity of the following argument

S_1: Thieves are jailed

S_2: Serious people read good books

S_3: Graduates are serious people

Conclusion: No graduate is thief (3 marks)

QUESTION TWO (20 MARKS)

a) Let $S = \{1,2,...,15\}$ and R be a relation on S defined by $(a,b)R(c,d)$ if and only if $ad = bc$.

i. Show that R is an equivalence relation (6 marks)

ii. Find the equivalence class of (1,2) (3 marks)

b) Let R be an equivalence relation on a set A. Prove that the following statements are equivalent;

i. aRb

ii. $[a] = [b]$

iii. $[a] \cap [b] \neq \phi$ (6 marks)

c) Prove the DeMorgan's law in Boolean algebra $(x+y)' = x'y'$ (5 marks)

QUESTION THREE (20 MARKS)

a) Use the principle of mathematical induction to prove the sum of a finite number of a geometric progression with an initial term a and a common ratio r. i.e. $\sum_{k=0}^{n} ar^k = \frac{ar^{k+1} - a}{r-1}$ where $r \neq 1$ and k is non negative integer (5 marks)

b) Let a=938 and b=1281. Use the division algorithm to find the gcd(a,b) and therefore find integers m and n such that d=am +bn (6 marks)

c) Solve the linear congruence equation $33x \equiv 31 (\bmod 280)$ (6 marks)

d) Prove the absorption law $x(x+y) = x$ using identities of Boolean algebra explaining each step. (3 marks)

QUESTION FOUR (20 MARKS)

a) Solve the congruence equation $f(x) = 34x^4 - 47x^3 + 54x^2 - 44x + 97 \equiv 0 (\bmod 8)$

(10 marks)

b) Consider the third order homogeneous recurrence relation $a_n = 2a_{n-1} + 3a_{n-2}$

i. Find the general solution (5 marks)

ii. Find the initial solution given $a_0 = 1 a_1 = 3,$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Prerequisites in a college are partial ordering of available classes. Denote A is a requisite of B by $A \prec B$. Let C be the ordered set of mathematics courses and their prerequisite as shown below.

Class	Math 101	Math 201	Math 250	Math 251	Math 340	Math 341	Math 450	Math 500
Prerequisite	None	Math 101	Math 101	Math 250	Math 201	Math 340	Math 201,250	Math 250,251

i. Draw with explanations the Hasse diagram for the partial ordering of the classes (3 marks)

ii. Find all the minimal and maximal elements of C and verify them (4 marks)

iii. Does C have a first or last element? Explain (4 marks)

c) Consider the Boolean algebra D_{120}

i. Find its elements and indicate their complements (4 marks)

ii. Find the set A of atoms (2 marks)

iii. State with reasons whether or not the following sets are sub-algebras

$X = \{1,2,6,120\}$ $Y = \{1,2,5,24,6,20,60,120\}$

(4 marks)
