CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION (SCIENCE), BACHELOR OF SCIENCE (ECOSTAT) , BACHELOR OF SCIENCE (MATHS, PHYSICS, COMPUTER SCIENCE), BACHELOR OF ARTS (ECON/MATH) AND BACHELOR OF SCIENCE (ELECTRONIC ENGINEERING)

MATH 124: GEOMETRY AND LINEAR ALGEBRA

STREAMS: BED (SCI,ARTS) BSC(ECON STAT,MATH,PHYS,COMP SCI) BA(ECON MATH), BSC (E.E) BSC (ACTUARIAL SCI) TIME: 2 HOURS

DAY/DATE: THURSDAY 13/12/2018 11.30 A.M – 1.30 P.M

INSTRUCTIONS

- **Answer question one and any other two questions**
- **Do not write on the question paper**

QUESTION ONE (COMPULSORY) (30 MARKS)

(a) Determine the equation of a circle whose diameter has end points (4,-1) and (-6,7). [3 marks]

(b) Find the equation of the ellipse which passes through the origin and has foci at (-1,1) and (1,1) [5 marks]

(c) Given A = $\begin{bmatrix} 2 & 3 & -3 \\ 2 & -1 & 2 \\ 2 & 4 & -4 \end{bmatrix}$ find A^{-1} [5 marks]

(d) Determine the area of the triangle whose verticals are at P(3,2,2) , Q (1,-1,2) and R(2,1,1) [3 marks]

(e) Given z = 4- 3i , determine a and b given. [3 marks]

$$\frac{\acute{z}}{z} = a + bi$$

(f) Convert the cartesian equation $x^2 - y^2 = (x^2 + y^2)^2$ into polar form [3 marks]

(g) Determine the shortest distance between the point (1,3) and the line whose equation is 3x -2y +5 = 0 [3 marks]

(h) (i) Find the co-ordinates of the focus F of the parabola $y^2 = 16x$. [2 marks]

(ii) Show that the point P(1,4) lies on the parabola $y^2 = 16x$.. [1 mark]

(iii) Calculate the distance PF for the parabola above. [2 marks]

QUESTION TWO (20 MARKS)

(i) Define the parabola. [2 marks]

(ii) Determine the equation of the parabola whose focus is (1,1) and directrix is y = -x -2. [5 marks]

(iii) Show that points (0,0) and (8,0) lies on the parabola in (a) (ii) above. [2 marks]

(b) Convert the polar equation $r^2 \sin 2\theta = 4$ into cartesian form. [3 marks]

(c) Two lines $L_1 \wedge L_2$ intersect at the point P. L_1 passes through (-4,0) and (0,6). If the equation of L_2 is y = 2x -2, determine the co-ordinates of P. [3 marks]

(d) Find the volume of a parallel piped whose edges are $\vec{A} = 2\underset{\sim}{i} - 3j + 4k$, $\vec{B} = \underset{\sim}{i} + 2j - k$ and $\vec{C} = 3\underset{\sim}{i} - j + 2k$.

[3 marks]

(e) Determine the modulus argument form of the complex number. $\underset{\sim}{Z} = -3 + 2\underset{\sim}{i}$. [2 marks]

QUESTION THREE (20 MARKS)

(a) (i) Define the ellipse. [2 marks]

(ii) Derive the equation of the ellipse whose centre is the origin, major axis is the y-axis and minor axis is the x-axis. [8 marks]

(b) Determine the angle between the vectors

a= $\underset{\sim}{i}$ -2$\underset{\sim}{j}$ +4$\underset{\sim}{k}$ and b= -4$\underset{\sim}{i}$ + j -2$\underset{\sim}{k}$. [3 marks]

(c) (i) Find the equation of a circle which passes through the points (7,1), (0,0) and (-1,7). [5 marks]

(ii) Determine the centre and radius of the circle in (c) (i) above. [2 marks]

QUESTION FOUR (20 MARKS)

(a) (i) Use the matrix inverse method to solve the system of equations.

2x +y +2z = 5

2y + 4x +3z = 9

2x +2y + z = 3 [9 marks]

(ii) Solve the system of equations in (a) (i) above using the Cramers's rule. [5 marks]

(b) Prove that $C^2 = A^2+B^2-2AB\cos\theta$ where $\vec{A}$, $\vec{B}$ and $\vec{C}$ are three sides of a triangle and θ is the angle between $\vec{A}$ and $\vec{B}$. [3 marks]

(c) Find the equation of the ellipse with semi-major axis 4 and eccentricity ½ if the centre is at the origin and major axis is horizontal. [3 marks]

QUESTION FIVE (20 MARKS)

(a) (i) Define the hyperbola. [2 marks]

(ii) Derive the equation of a hyperbola whose centre is at the origin and foci are along the x-axis. [8 marks]

(b) Find the graph of (analyze) the equation

x^2 - 4 y^2 -2x +16y – 14 = 0 [10 marks]
