

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS; BACHELOR OF SCIENCE (MATHEMATICS, ACTUARIAL SCIENCE, ELECTRONICS ENGINEERING, PHYSICS, CHEMISTRY); BACHELORS OF ARTS (MATHS-ECON), BACHELOR OF SCIENCE IN COMPUTER SCIENCE, BACHELOR OF SCIENCE APPLIED COMPUTER SCIENCE, BACHELORS OF SCIENCE IN ECONOMICS AND STATISTICS

MATH 122: BASIC MATHEMATICS

STREAMS: AS ABOVE

TIME: 2 HOURS

DAY/DATE: MONDAY 10/12/2018

8.30 AM – 10.30 AM

INSTRUCTIONS:

- Answer Question ONE and ANY other TWO Questions
- Do not write on the question paper

QUESTION ONE (30 MARKS)

a. Let $p =$ It is hot and $q =$ It is raining. Give the verbal translation for
 (i) $\sim p \wedge q$ (ii) $p \rightarrow q$ (2 marks)

b. Evaluate $\sum_{k=1}^8 (-1)^{k+1} 3k$ (2 marks)

c. Find the first five terms of the sequence $a_n = (1+i)^n$, $i = \sqrt{-1}$. Hence find the sum of the first 6 terms in the simplest form i.e. $a + ib$ (4 marks)

$$f(x) = \frac{4}{\sqrt{x^2 - x - 2}}$$

- d. Determine the domain and range for the function (3 marks)
- e. Let $z_1 = (5, 12^0)$, $z_2 = (2, 23^0)$, show that $z_1 z_2 = (7, 35^0)$ (4 marks)
- f. Find the number of ways that 3 girls and 5 boys can be seated in a row such that none of the 3 girls sit next to the other (4 marks)

$$\left(2x + \frac{1}{5x^7}\right)^{10}$$

- g. Find the constant term in the expansion of (3 marks)
- h. Consider the following argument:
 S_1 : Poets are happy people.
 S_2 : Every doctor is wealthy.
 S_3 : No happy person is wealthy.

Determine the validity of the conclusion that "Doctors are happy people". (3 marks)

$${}^n P_5 = 20 \times {}^n P_3$$

- i. Solve for n given that (3 marks)

- j. Given that $\cos \theta = \frac{12}{13}$ and $\sin \phi = \frac{4}{5}$ where θ is obtuse and ϕ is acute, calculate without using tables or calculators the values of $\cot(\theta - \phi)$ (3 marks)

QUESTION TWO 20 MARKS

- a. The first term of an arithmetic sequence is 2. The first term of a geometric sequence is also 2 and its common ratio equals the common difference of the arithmetic sequence. The third term of the geometric sequence exceeds the square of the first term of the arithmetic sequence by 124. Find:
 i. The common difference (3 marks)
 ii. The sum of the first 10 terms of the arithmetic sequence (2 marks)

- b. Simplify the following expression

$$\frac{1+i}{1-i} - (1-2i)(2+2i) + \frac{3-i}{1+i} \quad (5 \text{ marks})$$

$$f(x) = \frac{1}{2}x + 4 \quad (f \circ g)(x) = \frac{7x-1}{2x} \quad g(x)$$

- c. Given that and , find (5 marks)

QUESTION THREE (20 MARKS)

a. Use mathematical induction to prove that $3+6+9+\dots+3n=\frac{3n(n+1)}{2}$ (5 marks)

b. Given the sets $A=\{2,5,10,19,51\}$, $B=\{3,9,17,13,26\}$,

$$C=\{9,17,33\}, D=\{1,7\} \wedge \varepsilon=\{2,3,5,7,9,10,17,19,26,33,51,100\}$$

(i) Draw the venn diagram for the sets (3 marks)

(ii) What is $(A \cup B) \cap C^c$? (2 marks)

(iii) Give the power set of D (2 marks)

c. Use analytic method to prove that if ε is the universal set and A, B, C are sets in ε , then (i) $A^c \cup B^c = (A \cap B)^c$ (4 marks)

(ii) $A - (B \cup C) = (A - B) \cap (A - C)$ (4 marks)

QUESTION FOUR (20 MARKS)

a. Prove the following

(i) $\frac{\cos^2 \theta}{1 + \sin \theta} + \frac{\sin^2 \theta}{1 - \sin \theta} = 2$ (3mks)

(ii) $2 \csc 2x + \tan x = \sec^2 x$ (3mks)

b. Solve $3 \sin \theta - \cos \theta = 1$, $0 \leq \theta \leq 360$ (4mks)

c. Expand $(1+x)^{\frac{1}{2}}$ upto the 4th term, Hence use the expansion to approximate $\sqrt{5}$ (5 marks)

- d. Find the modulus and the conjugate of $z = \frac{3-i}{\sqrt{2}+3i}$ (5 marks)

QUESTION FIVE (20 MARKS)

- a. Prove that $\sqrt{2}+\sqrt{3}$ is an irrational number. (4 marks)

- b. By means of truth tables show that
 $p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$ (3 marks)

- c. Use logics to determine the validity of the following statement
 “if girls are blonde, they are popular with boys. Ugly girls are unpopular with boys.
 Intelligent girls are ugly. Therefore blonde girls are not intelligent.” (6 marks)

- e. Each of the 100 students in first year faculty of science studies at least one of the subsidiary subjects: Mathematics, Programming and Accounting. Given that 65 study Mathematics, 45 study programming, 42 study accounting, 20 study Mathematics and programming, 25 study mathematics and accounting and 15 study programming and accounting, find the number that study:

- i. All the three subsidiary subjects
 - ii. Mathematics and accounting but not programming
 - iii. Only accounting as a subsidiary subject (7 marks)
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