CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF EDUCATION AND BACHELOR OF ARTS

MATH 210(201): LINEAR ALGEBRA I

STREAMS: BSC, BED (ARTS) **TIME: 2 HOURS**

DAY/DATE: MONDAY 01/2/2021 **8.30 A.M – 10.30 A.M**

INSTRUCTIONS:

- Answer question **ALL** the questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write on the question paper
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

(a) Solve the following

$(a)\ ex = \pi\ (b)\ 3x - 4 - x = 2x + 3\ \ (c)\ 7 + 2x - 4 = 3x + 3 - x$ (6 marks)

(b) Consider the system in unknowns $x\ and\ y$

$$x - ay = 1$$
$$ax - 4y = b$$

Find which values of a does the system have a unique solution, and for which pairs of values (a, b) does the system have more than one solution. (5 marks)

(c) Show that the determinant of the matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given by $a_{11}a_{22} - a_{12}a_{21}$ (3 marks)

(d) (i) When is a matrix A said to be in Echelon form? (2 marks)

(ii) Determine the rank of the following matrix by reducing the matrix to echelon form

$$A = \begin{pmatrix} 6 & 3 & 1 & 4 \\ 2 & 1 & 3 & 0 \\ 4 & 2 & 6 & 0 \\ 6 & 3 & 1 & 4 \end{pmatrix}$$ (4 marks)

(e) Determine if $S = \{(1,1), (,1-1)\}$ is a basis for R^2 (5 marks)

(f) Evaluate the WROSKIAN $w(sinx, cosx, sin2x, \frac{3}{4}\pi)$ (3 marks)

(g) Let $V = \mathbb{R}^3$ and $W = \{(a, b, 0): a, b \in \mathbb{R}\}$. Determine if W is a subspace of $\mathbb{R}^3$. (2 marks)

QUESTION TWO: (20 MARKS)

(a) Determine the value of `w' so that following system of equations has

(i) Unique solution

(ii) Infinitely many solutions.

(iii) No solution

$x_1 \qquad - 3x_3 = -3$

$2x_1 + ax_2 - x_3 = -2$

$x_1 + 2x_2 + ax_3 = 1$ (8 marks)

(b) Find the determinant of matrix A by first partitioning the matrix

A = $\begin{pmatrix} 3 & 6 & 9 & 3 \\ -1 & 0 & 1 & 0 \\ 1 & 3 & 2 & -1 \\ -1 & 2 & -1 & 1 \end{pmatrix}$ such that $A_{11} = \begin{pmatrix} 3 & 6 \\ -1 & 0 \end{pmatrix}$, $A_{12} = \begin{pmatrix} 9 & 3 \\ 1 & 0 \end{pmatrix}$, $A_{21} = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$,

$A_{22} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$ (6 marks)

(c) Determine if $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined as $T(x_1, x_2) = (2x_1, x_1 - x_2, x_2 + 2x_1)$ is a linear transformation. (6 marks)

QUESTION THREE: (20 MARKS)

a) Solve the system of equation using Cramer's Rule

$x_1 + 2x_2 - x_3 = 2$

$3x_1 + 6x_2 + x_3 = 1$

$3x_1 + 3x_2 + 2x_3 = 3$ (6 marks)

b) Let V = R^3 and S = {(2, 3, 5), (1, 2, 4),(-2, 2, 3)}. Determine if (10, 1, 4) ∈ L(S), where L(S) is a subset of V . (6 marks)

c) Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be defined as $(x) = \begin{pmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

Find the Basis for $R(T)$ and the Nullity (T). (8 marks)
