CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION (ARTS)

MATH 447: APPLIED MULTIVARIATE ANALYSIS

STREAMS:BSC,BED, BA **TIME: 2 HOURS**

DAY/DATE: FRIDAY 24/09/2021 **8.30 A.M – 10.30 A.M**

INSTRUCTIONS

Answer Question ONE and any other TWO Questions

QUESTION ONE (30 MARKS)

(a) Outline five objectives of multivariate analysis [5marks]

(b) Let $\bar{X} \sim N(\underline{\mu}, \Sigma)$ with $\underline{\mu'} = [44\ 20\ 16]$ and $\Sigma = \begin{bmatrix} 64 & 8 & -\frac{16}{5} \\ 8 & 16 & 4 \\ -\frac{16}{5} & 4 & 4 \end{bmatrix}$

Find,

i. the distribution of $Y = \begin{pmatrix} X_1 - X_3 \\ X_1 + 2X_2 + X_3 \end{pmatrix}$ [5marks]

ii. the standard deviation matrix $V^{\frac{1}{2}}$ and the correlation matrix [3marks]

iii. the distribution of X_1 given that $X_2 = 15$ and $X_3 = 18$ [5 marks]

iv. the regression function of X_3 on X_1 and X_2 [3 marks]

v. the partial correlation coefficient between X_1 and X_3 for fixed values of X_2 [3marks]

(c) Given below is a data matrix

$$X = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 0 \\ 5 & 6 & 4 \end{bmatrix}$$

Find

i. the generalized sample variance
ii. the total sample variance [6marks]

QUESTION TWO (20 MARKS)

Observations on three responses are collected for two treatments. The observation vectors are as given below.

Treatment	A	A	A	A	A	B	B	B
X_1	8	9	5	4	4	6	7	5
X_2	15	14	12	9	10	6	8	10
X_3	4	5	4	4	3	9	8	7

Find

i. The matrix of sum of squares due to treatment [7marks]

ii. The matrix of residual sum of squares [3marks]

iii. the Wilk's lambda statistics and use it to test the hypothesis that there is no treatment effect at 5% significance level [6marks]

iv. State all the assumptions of MANOVA [4marks]

QUESTION THREE (20 MARKS)

(a) Let data array X have the covariance matrix. $\Sigma = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Find the Eigen values and Eigen vectors of Σ and Σ^{-1} [5marks]

(b) A random sample of size 10 was taken from a bivariate normal and sample mean and inverse of the sample variances were obtained as

$$\underline{\bar{x}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \text{ and } S^{-1} = \begin{bmatrix} 0.1890 & -0.099 \\ -0.099 & 0.266 \end{bmatrix}$$

Test the hypothesis $H_0: \underline{\mu} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ versus $H_1: \underline{\mu} \neq \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ at 5% significance level. [10marks]

(c) Let $\boldsymbol{P} = \frac{1}{13}\begin{pmatrix} 5 & 12 \\ -12 & 5 \end{pmatrix}$ and $\boldsymbol{Q} = \begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix}$ Show that;

i. $\boldsymbol{P}$ is an orthogonal matrix [3marks]

ii. $\boldsymbol{Q}$ is positive definite [2marks]

QUESTION FOUR (20 MARKS)

(a) (i)Verify the relationships

$$\boldsymbol{V}^{\frac{1}{2}}\boldsymbol{\rho}\boldsymbol{V}^{\frac{1}{2}} = \boldsymbol{\Sigma} \quad \text{and} \quad (\boldsymbol{V}^{\frac{1}{2}})^{-1}\boldsymbol{\Sigma}\,(\boldsymbol{V}^{\frac{1}{2}})^{-1} = \boldsymbol{\rho}$$

(ii) Given

$$\boldsymbol{\rho} = \begin{bmatrix} 1 & \frac{1}{6} & \frac{1}{5} \\ \frac{1}{6} & 1 & -\frac{1}{5} \\ \frac{1}{5} & -\frac{1}{5} & 1 \end{bmatrix} \text{ and } \boldsymbol{V} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

Determine $\boldsymbol{\Sigma}$ [10marks]

(b) Given the following data from a bivariate normal distribution

$$X = \begin{pmatrix} 24 & 22 & 20 & 17 & 27 & 21 & 22 & 19 & 17 \\ 21 & 24 & 29 & 24 & 22 & 20 & 25 & 26 & 25 \end{pmatrix}$$

Test the hypothesis that *Ho*: $\mu = [24 \quad 19]$ against *H₁*: $\mu \neq [24 \quad 19]$ at 5% significance level

[10marks]

QUESTION FIVE (20 MARKS)

The random vector $\underline{\boldsymbol{X'}} = [\boldsymbol{X_1}\ \boldsymbol{X_2}\ \boldsymbol{X_3}\ \boldsymbol{X_4}\ \boldsymbol{X_5}]$ with mean vector $\underline{\boldsymbol{\mu_x}} = [\mathbf{2\ 4 - 1\ 3\ 0}]$ and the variance- covariance matrix

$$\boldsymbol{\Sigma} = \begin{bmatrix} 4 & -1 & 1/2 & -1/2 & 0 \\ -1 & 3 & 1 & -1 & 0 \\ 1/2 & 1 & 6 & 1 & -1 \\ -1/2 & -1 & 1 & 4 & 0 \\ 0 & 0 & -1 & 0 & 2 \end{bmatrix}$$

Partition $\underline{\boldsymbol{X'}} = [\boldsymbol{X_1}\ \boldsymbol{X_2}\ \boldsymbol{X_3}\ \boldsymbol{X_4}\ \boldsymbol{X_5}]$ into $\underline{\boldsymbol{X'}} = [\underline{\boldsymbol{X'_1}}\ \underline{\boldsymbol{X'_2}}]\,'$ where $\underline{\boldsymbol{X'_1}} = [\boldsymbol{X_1}\ \boldsymbol{X_2}]$ and

$\underline{\boldsymbol{X'_2}} = [\boldsymbol{X_3}\ \boldsymbol{X_4}\ \boldsymbol{X_5}]$. Let $\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{1} & \mathbf{1} \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{-2} \end{bmatrix}$. [20marks]

Determine

i. $\boldsymbol{E}(\underline{\boldsymbol{X_1}})$

ii. $\boldsymbol{E}(\boldsymbol{A}\underline{\boldsymbol{X_1}})$

vi. $\boldsymbol{COV}(\underline{\boldsymbol{X_2}})$

vii. $\boldsymbol{COV}(\boldsymbol{A}\underline{\boldsymbol{X_1}})$

iii. $E(\underline{X_2})$

iv. $E(\underline{BX_2})$

v. $COV(\underline{X_1})$

viii. $COV(\underline{BX_2})$

ix. $COV(\underline{X_1}, \underline{X_2})$

x. $COV(\underline{AX_1}, B\underline{X_2})$
