CHUKA   UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

ACMT 404: DEMOGRAPHIC TECHNIQUES

STREAMS: BSC   **TIME: 2 HOURS**

DAY/DATE: THURSDAY 23/09/2021   **8.30 A.M – 10.30 A.M.**

INSTRUCTIONS:

QUESTION ONE (30 MARKS)

A. List and explain two theories of population trends.   (4 marks)

B. Define what graduation is. List and explain three aims of graduation   (5 marks)

C. List and explain advantages and disadvantages of graduation by mathematical formula.

                                                                                 (6 marks)

D. Suppose you have been given the following graduated rates. Test for smoothness.

X	$q_{\overline{x}}^{0}$
40	0.001056
41	0.001167
42	0.001290
43	0.001426
44	0.001576
45	0.001742

                                                                                 (6 marks)

E. Define the below terms
   i.   Temporary initial selection
   ii.   Self-selection
   iii.   Reverse selection
   iv.   Adverse selection   (4 marks)

F. A graduation covers 20 age groups and has resulted in 6 positive and 14 negative deviations. Carry out a signs tests on these data   (5 marks)

QUESTION TWO (20 MARKS)

a) The estimated population (female) for Kijabe town ages 78 to 81 last birthday on 1st January, 1989 was as follows.

Age last birthday	estimated female population
78	2050
79	2026
80	2021
81	2021

It is estimated that in each year's 1989 to 1990, there will be a net inward migration of 170 women each of the ages 78 and 79 last birthday at the date of moving to the town and 200 women at each of the ages 80 and 81 last birthday at the date of moving to the town. The mortality rate of woman in this town at ages 78-81 is a follows:

x	q_x
78	0.053
79	0.059
80	0.065
81	0.072

Estimate the population of each ages 80 and 81 last birthday on

   i.   1st January 1990
   ii.   1st January 1991   (10 marks)

b) List and explain simple models of patterns of population growth (5 marks)

c) List the sources of errors in fertility data (5 marks)

QUESTION THREE (20 MARKS)

A) (i) Name three defects of graduation which the x^2 test might fail to detect. (3marks)

(ii) The following data and graduated rates of mortality (indicated by ($q\frac{0}{x}$) ref__ to a certain mortality investigation.

Age x	O	E_x	q_x	θ_x-E_xq_x	$z_x = \dfrac{\theta_x\text{-}E_xq_x}{\sqrt{E_xq^o{}_x(1\text{-}q_x)}}$
70	8	673	0.01666	-3.2	-0.964
71	15	747	0.01823	+1.1	+0.059
72	10	701	0.02077	-4.6	-1.218
73	12	830	0.02318	-7.2	-1.661
74	13	614	0.02587	+2.9	-0.737
75	21	669	0.02886	+1.7	+0.0393
76	19	786	0.03522	-6.3	-1.273
77	21	655	0.03540	-2.5	-0.525
78	24	714	0.04002	-4.6	-0.878
79	32	623	0.04461	+4.2	+0.815

In respect to ONE of the defects in (1) above apply the suitable test to test this data using 10% significance level (7 marks)

B) Suppose we are carrying out an investigation on 70 year old between 01/01/2013 and 01/01/2014. We have one individual born 01/11/1942, join the investigation on 01/03/2013 and dies 01/08/2013. And another born 01/09/1942 who joined the investigation when it started and he is still alive when it ended. Calculate the total initial exposed to risk (5 marks)

C) List and explain sources of demographic data (5 marks)

QUESTION FOUR (20 MARKS)

A) Explain three types or categories of rates used in demographic techniques (6 marks)

B) Let P(t) denote the estimated population of a certain country at time t years from the present

A demographer believes that P(t) will satisfy the logistic differential equation

$$\frac{dP(t)}{dt} = \rho P(t) - K[P(t)]^2 \quad t \geq 0$$

Where ρ and k, are known positive constants. The present population estimate p(0) is known as P(0)< ρ /k.

    i.   By making the substitution $\mu(t) = [P(t)]^{-1}$, find the first order linear differential equation for $\mu(t)$).   (3 marks)

    ii.   Solve for the u(t) and hence find a formula for P(t) in terms of P(0), P and K. (4 marks)

    iii.   What is the limit as $t \rightarrow \infty$ for the estimated population   (4 marks)

    iv.   The demographer has now decided to reverse her model. The logistics equation is to be replaced by the equation

$$\frac{dP(t)}{dt} = \rho P(t) - Ke^{-\rho t}[P(t)]^2 \quad (t \geq 0)$$

                    (4 marks)

       Find the expression for P(t) in terms of P(0)   (3 marks)

QUESTION FIVE (20 MARKS)

A. List and explain strengths and weakness of census data.   (8 marks)

B. Differentiate between crude death rate (CDR) and specific death rates (SDR) (4 marks)

C. List and explain two techniques of population projection.   (4 marks)

D. List and explain two methods closed in gradation by parametric formula.   (4 marks)