EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN ECONOMICS

## MSEC 831:MATHEMATICAL METHODS FOR ECONOMISTS

STREAMS:
TIME: 3 HOURS

DAY/DATE: THURSDAY 15/07/2021
INSTRUCTION

## Answer Question ONE and any other two questions

## QUESTION ONE

(a) The production function for a firm is given as $\mathrm{Q}=200-L^{2}-K^{2}$

The corresponding cost function is given as $\mathrm{C}=2 \mathrm{~L}+3 \mathrm{~K}$
The firm wishes to maximize its output for a specified level of outlay of $\mathrm{C}=30$.
You are required to:
(i) Write a maximization problem for the firm.
(ii) Determine the level of K and L for which Q is maximized.
(iii) Determine whether the second order condition is satisfied.
(iv) Determine the maximum level of Q .
(b) Given the following production function $\mathrm{Q}=C K^{\alpha} L^{1-\alpha}$

Required:
(i) Express MPL in terms of $\mathrm{Q}, \alpha$ and L
(ii) Express MPK in terms of $\mathrm{Q}, \alpha$ and K
(iii) $\quad$ Show that $\mathrm{Q}=\mathrm{K}(\mathrm{MPK})+\mathrm{L}(\mathrm{MPL})$
(c) The ATC and the AR of the frim are given as
$2-\frac{4}{Q}=Q-A T C$
$4 \mathrm{Q}=2-\mathrm{AR}$
(i) Find the price elasticity of demand at $\mathrm{P}=4$. [1 mark]
(ii) Show what level of Q is ATC at minimum. [1 mark]
(iii) Determine the point at which is ATC $=$ MC. [2 marks]
(iv) Discuss your results in ii and iii above. [2 marks]
(v) Find the profit maximizing Q if a tax of 2 per unit is imposed. [3 marks]
(vi) Find the profit maximizing Q if a subsidy of per unit is imposed. [3 marks]
(d) A three sector input model is given by the following.

|  | 1 | 2 | 3 | D | X |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | $X_{11}$ | $X_{12}$ | $X_{13}$ | $D_{1}$ | $X_{1}$ |
| 2 | $X_{21}$ | $X_{22}$ | $X_{23}$ | $D_{2}$ | $X_{2}$ |
| 3 | $X_{31}$ | $X_{32}$ | $X_{33}$ | $D_{3}$ | $X_{3}$ |
| V | $V_{1}$ | $V_{2}$ | $V_{3}$ | GNP |  |
| X | $X_{1}$ | $X_{2}$ | $X_{3}$ |  |  |

## Required:

(i) If the input -output coefficient are denoted by $\alpha_{i j}$ where (ij) $=1,2,3$ ) write these inputs - output coefficients in terms of $x_{i j}$ where (ij) $=1,2,3$ ) [4 marks]
(ii) Find GNP by expenditure approach.
[2 marks]
(iii) Find GNP by factor input approach.
[2 marks]

## QUESTION TWO

(a) Given the following market model,
$Q_{d}=\alpha_{0}-\alpha_{1} P$
$Q_{s}=\beta_{0}+\beta_{1} P$
(i) Find the total differential of the following function.
$\mathrm{Z}=2 X_{1}^{2}+3 X_{1} X_{2}+5 X_{2}^{2}$
[4 marks]

## QUESTION THREE

(a) Using well labelled diagrams. Shows the difference between consumer surplus and producers surplus.
[10 marks]
(b) Given the following supply function, $Q^{2}+Q+2-P=0$ Find the producers' surplus when $\mathrm{Q}=2$
(c) Given the following demand function: determine consumer' surplus

$$
\mathrm{Q}+\mathrm{P}=8 \quad \text { where } \mathrm{P}=3
$$

## QUESTION FOUR

A national income model is given by the following equations.
$\mathrm{Y}=\mathrm{C}+I_{0}+G_{0}$
$\mathrm{C}=\alpha_{1}+\alpha_{2} Y^{d}$
$Y^{d}=Y-T$
$\mathrm{T}=\lambda Y$
(a) Find the equilibrium $\mathrm{Y}, \mathrm{C}$ and T
(c)Determine the following multipliesr and interpret your results for each.
(i) Investment multiplier
(ii) Government expenditure multiplier
[2 marks]
(iii)Autonomous consumption multiplier.

