

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

**SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF  
SCIENCE MATHEMATICS**

MATH 411: DIFFERENTIAL GEOMETRY

STREAMS: BSC. MATH

TIME: 2 HOURS

DAY/DATE: TUESDAY 21/09/2021

2.30 P.M. – 4.30 P.M.

**INSTRUCTIONS:**

- Answer Question **ONE** and any other **TWO** Questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

**QUESTION ONE (30 MARKS)**

- a) i) Define a regular representation function on an interval  $I$ . Hence show that the function  $\mathbf{x} = (3t + 1)\mathbf{e}_1 + (t^4 + 5)\mathbf{e}_2$ ,  $-\infty < t < \infty$  is a regular parametric representation.

(3 marks)

- (ii) When is a real valued function  $t = t(\theta)$  on an interval  $I_\theta$  said to have an allowable change of parameter? Take  $t = (b - a)\theta + a$ ,  $0 \leq \theta \leq 1$ ,  $a < b$  to illustrate this.

(4 Marks)

- b) Find the equation of the oscillating plane of the helix  $\mathbf{x} = (\cos t)\mathbf{e}_1 + (\sin t)\mathbf{e}_2 + t\mathbf{e}_3$  at  $t = \frac{\pi^c}{2}$

(6 marks)

- c) Compute the arc length to the curve  $\mathbf{x} = 3(\cosh 2t)\mathbf{e}_1 + 3(\sinh 2t)\mathbf{e}_2 + 6t\mathbf{e}_3$

$$0 \leq t \leq \pi$$

(3 Marks)

- d) Along the helix  $= (a \cos t)e_1 + (a \sin t)e_2 + bte_3, a > 0, b \neq 0$ . Find
- (i) the unit tangent vector  $\mathbf{t}$ . (3 marks)
  - (ii) the curvature vector  $\mathbf{k}$  (3 marks)
  - (iii) the radius of curvature (2 marks)
- e) (i) State without proof the Fundamental existence and uniqueness Theorem (2marks)
- (ii) Show that the First Fundamental form is positive definite. (3 marks)

**QUESTION TWO (20 Marks)**

- a) Define torsion  $\tau(s)$  of the curve  $C$  at the point  $\mathbf{x}(s)$ . Hence show that the sign of  $\tau$  is independent of the sense of principal normal vector  $\mathbf{n}$  and the orientation of  $C$ . (6 marks)
- b) State and prove the Serret-Frenet equations of a curve (8 marks)
- c) Find the equations of the tangent line and normal plane to the curve  $\mathbf{x} = t\mathbf{e}_1 + t^2\mathbf{e}_2 + t^3\mathbf{e}_3$  at  $t = 1$  (6 marks)

**QUESTION THREE (20 Marks)**

- a) Find the equations of the tangent line and normal plane to the curve  $\mathbf{x} = t\mathbf{e}_1 + t^2\mathbf{e}_2 + t^3\mathbf{e}_3$  at  $t = 1$  (6 marks)
- b) Find the curvature and torsion of the curve  $\mathbf{x} = (3t - t^3)\mathbf{e}_1 + 3t^2\mathbf{e}_2 + (3t + t^3)\mathbf{e}_3$  and compare your results. (7 marks)
- c) When do we say an arc  $\mathbf{x} = \mathbf{x}(t), a \leq t \leq b$  said to be rectifiable? Hence or otherwise show that the arc  $\mathbf{x} = t\mathbf{e}_1 + t^2\mathbf{e}_2, 0 \leq t \leq 1$  is rectifiable. (7 marks)

**QUESTION FOUR (20 Marks)**

- a) i) Derive the First Fundamental form  $I$  to the coordinate patch  $\mathbf{x} = \mathbf{x}(u, v)$  on a surface of class  $\geq 2$ . (7 marks)
- ii) Hence prove that the First Fundamental form depends only on the surface and not on the particular representation (7 marks)
- b) Consider the surface represented by  $\mathbf{x} = u\mathbf{e}_1 + v\mathbf{e}_2 + (u^2 - v^2)\mathbf{e}_3$ . Find its second fundamental form  $II$  (6 marks)

**QUESTION FIVE (20 Marks)**

a) Prove that along a regular curve  $\mathbf{x} = \mathbf{x}(s)$ , the curvature  $|\mathbf{k}| = \frac{|\mathbf{x}' \times \mathbf{x}''|}{|\mathbf{x}'|^3}$  (7 marks)

b) Distinguish an involute and an evolute of a curve  $C$ . Hence show that the curvature of an

involute  $\mathbf{x}^* = \mathbf{x} + (c - s)\mathbf{t}$  of  $\mathbf{x}(s)$  is given by  $k^{*2} = \frac{k^2 + \tau}{(c-s)^2 k^2}$

(13 marks)

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