CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATIONS FOR THE AWARD OF BACHELOR OF SCIENCE IN APPLIED COMPUTER SCIENCE.

MATH 407: FOURIER ANALYSIS **TIME: 2 HOURS**

INSTRUCTIONS

Answer question one and any other two questions

Adhere to the instructions on the answer booklet.

QUESTION ONE Compulsory.

a. Find the complex Fourier series for $f(x)=\begin{cases}0, -\pi \le x \le 0\\ 2, 0 \le x \le \pi\end{cases}$ and show that $c_n = \begin{cases}\dfrac{1}{in\pi}, n \text{ is odd}\\ 0, n \text{ is even}\end{cases}$ 5mks

b. Evaluate $\int_0^1 \left(1-x^3\right)^{-\frac{1}{2}} dx$ using the Beta function 5mks

c. Using the Fourier cosine integral representation of an appropriate function, show that $\int_0^\infty \frac{\cos wx}{k^2+w^2}dw = \frac{\pi e^{-kx}}{2k}$ 5mks

d. Obtain the Fourier sine transform of $f(x)=\frac{e^{-3x}}{x}$ 5mks

e. Determine the exponential form of the Fourier series for the function defined by $f(t)=2t,\ -\pi \le t \le \pi$ and show that the Fourier series is equivalent to $f(t)=4\left(\sin t-\frac{1}{2}\sin 2t+\frac{1}{3}\sin 3t-\frac{1}{4}\sin 4t+...\right)$ 5mks

f. Evaluate $\int_0^\infty x^{n-1}e^{-4x^2}dx$ by the Gamma function 5mks

QUESTION TWO

a. State the Dirichlets conditions for a Fourier series 5mks

b. Find the Fourier series representing $f(x)=x, 0<x<2\pi$ 7mks

c. Obtain a_0, a_n *and* b_n for the periodic function $f(x)=\begin{cases}0, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$ 8mks

QUESTION THREE

a. A periodic function of period 4 is defined as $f(x)=\begin{cases}x, & 0<x<2\\ -x, & -2<x<0\end{cases}$

Obtain a_0, a_n *and* b_n 8mks

b. Use the Fourier sine series for $f(x)=1$, in $0<x<\pi$ to show that $\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}......$

7mks

c. Evaluate $\int\limits_0^\infty \sqrt[4]{x}e^{-\sqrt{x}}\,dx$ 5mks

QUESTION FOUR

a. Find the Fourier cosine transform of

$$f(x)=e^{-2x}+4e^{-3x}$$

6mks

b. Solve for f(x) from the integral equation $\int\limits_0^\infty f(x)\cos sx\,dx=e^{-s}$ 7mks

c. Use Parsaval`s identity to show that $\int\limits_0^\infty \frac{dx}{\left(x^2+1\right)^2}dt=\frac{\pi}{4}$ 7mks

QUESTION FIVE

a. Solve $U_t=kU_{xx}$ *for* $x\geq 0, t\geq 0$, under the given conditions $U=U_0$ *at* $x=0, t>0$, with initial conditions $U(x,0)=0, x\geq 0$ by Fourier transforms. 8mks

b. Find the finite Fourier sine and cosine transform of $f(x)=x$, *in* $(0,l)$ 6mks

c. Find the finite Fourier sine transform of $f(x)=1$ *in* $(0,\pi)$ and use the inversion theorem to prove that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+...=\frac{\pi^2}{8}$$ 6mks