CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION (SCIENCE,ARTS) , BACHALOR OF SCIENCE IN MATHEMATICS, BACHELOR OF ARTS (MATH -ECONS)

MATH 401: TOPOLOGY I

STREAMS: **TIME: 2 HOURS**

DAY/DATE: MONDAY 20/09/2021 **2.30 P.M – 4.30 P.M**

INSTRUCTIONS

Answer Question ONE and ANY Other TWO Questions.
Do not write on the question paper.

QUESTION ONE: (30 MARKS)

(a) Distinguish the following terms as used in topology

(i) An indiscrete topology and a Sierpinski's topology

(ii) An accumulation point and an interior point p of the subset A of a topological space (X,τ)

(iii) A first category and second category subset A of a topological space (X,τ).

(iv) A regular topological space and a perfect topological space

(v) A base and a local base for the topology τ (10marks)

(b) Let (X,τ) be a topological space. Prove that a arbitrary intersection of closed sets is also closed (3marks)

(c) Prove that if A is a subset of a discrete topology, then set of its derived points is empty. (4marks)

(d) Consider the following topology on $X = \{a, b, c, d, e\}$and

$\tau = \{\{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X, \emptyset\}$.

If $B = \{a, e\}$. Show that the boundary of B, $\delta B = \bar{B} \cap \overline{X/B}$ (5marks)

(e) Determine the neighborhood system of a point p in an indiscrete space X (3marks)

(f) Show that for a subbases θ for a topology τ on X and a subset A of X, the class $\theta_A = \{A \cap b : b \in \theta\}$ is a subbase for the relative topology τ_A on A (5marks)

QUESTION TWO: (20 MARKS)

(a) (i) Using a counter example show that an open function or a closed function need not be continuous (3marks)

(ii) Let $f: X \to Y$ and $g: Y \to Z$ be continuous functions. Prove that the composite function $g \circ f$ is continuous (3marks)

(b) Prove that a point p of a non-empty set X is an accumulation point of $A \subset X$ if and only if every member of some local base β_p at the point p contains a point of A different from p (7marks)

(c) Let $P: X \to Y$ be an open map and let $S \subset Y$ be any subset of Y and A is a closed set in X such that $P^{-1}(S) \subset A$. Show that $S \subset B$ and $P^{-1}(B) \subset A$. (7marks)

QUESTION THREE: (20 MARKS)

(a) (i) Prove that a set G is open if and only if it is a neighborhood of each of its points (4marks)

(ii) Prove that a topological space X is a T_1 space iff every singleton subset $\{p\} \subset X$ is closed. (5marks)

(b) Let β be a class of subsets of a non-empty set X. Prove that β is a base for some topology on X iff

(i) $X = \cup \{B : B \in \beta\}$

(ii) For any $B, B^* \in \beta$, $B \cap B^*$ is the union of members of β (11marks)

QUESTION FOUR: (20 MARKS)

(a) Let $X = \{a, b, c, d, e\}$. Denote $\beta = \{\{a\}, \{b, c\}, \{d, e\}, \emptyset\}$. Show that β forms a base for a topology τ on X. Hence find also the topology τ. (5marks)

(b) Let A be a subset of a topological space(X, τ). Prove that τ_A is a topology on A, where $\tau_A = \{A \cap G : G \in \tau\}$ (8marks)

(c) Using appropriate counter examples show that a T_2 space $\Rightarrow T_1$ space and T_1 space $\Rightarrow T_0$space but a T_0 space $\nRightarrow T_1$ and a T_1 space $\nRightarrow T_2$ (7marks)

QUESTION FIVE: (20 MARKS)

(a) Let $g: X \to Y$ be a bijective and A a subset of X, prove that the following statements are equivalent.

(i) g is a homomorphism

(ii) g is open

(iii) g is closed

(iv) $g(\bar{A}) = \overline{g(A)}$ (12marks)

(b) Let $A \subset X$, where X is a non-empty topological space. Prove that $\bar{A} = \delta A \cup A^0$ (8marks)
