CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR AWARD OF DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND BACHELOR OF SCIENCE IN EDUCATION

MATH 326: METHODS OF APPLIED MATHS I

STREAMS: BSc. MATHS, BSc. ED **TIME: 2 HOURS**

DAY/DATE: WEDNESDAY 14/07/2021 **2.30 P.M. – 4.30 P.M.**

INSTRUCTIONS

- ***Answer question one and any two questions***
- ***Adhere to the instructions on the answer booklet.***

QUESTION ONE Compulsory

a. Find the recurrence relation satisfied by coefficients in the series solution of the differential equation $y'' + x^2 y = 0$, about the point x = 0 and obtain a_4 *and* a_6 (5 marks)

b. Solve in series the differential equation $y' - y = 0$, about the point x = 0 (5 marks)

c. Identify the nature of the singular points of the equation

$$x(x-2)^2 y'' + 2(x-2)y' + (x+3)y = 0$$ (5 marks)

d. Given the function $f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$, Obtain a_0 *and* a_n (5 marks)

e. Obtain a_n , for the Fourier series represented by $f(x) = e^x$, as a cosine Fourier series over (0, 1) (5 marks)

f. Find the Laplace transform of $\dfrac{\sin 2t}{t}$ (5 marks)

QUESTION TWO

a. Prove that the Laplace transform of $L(e^{at}) = \frac{1}{s-a}, \; s > a$ (5 marks)

b. Find the sine Fourier series for the function $f(x) = 1, \; in \; 0 < x < \pi$ (5 marks)

c. Find the Laplace transform of the following

i. $t^2 \cos 3t$ (5 marks)

ii. $te^{-t} \sin 2t$ (5 marks)

QUESTION THREE

a. Solve in series the differential equation, $\left(1-x^2\right)y'' - 2xy' + 2y = 0$ about the point x = 0 (10 marks)

b. Given the differential equation $3xy'' + 2y' + y = 0$, about the point x = 0.

i. Obtain the indicial equation of the differential equations and suggest a general solution to the equation. (6 marks)

ii. Find the recurrence relation satisfied by coefficients in the series solution of the differential equation and obtain a_1 (4 marks)

QUESTION FOUR

a. Given the function $f(x) = x, \; 0 \le x \le 2\pi$, Obtain the Fourier constants $a_0, \, a_n \; and \; b_n$ (6 marks)

b. Find a Fourier series to represent $f(x) = x^2, \; -\pi \le x \le \pi$ (6 marks)

c. Find the inverse Laplace transform of $\frac{1}{s^2 - 9}$ (3 marks)

d. Using the Laplace transforms, to evaluate $\int_0^\infty te^{-3t} \sin t \, dt$ (5 marks)

QUESTION FIVE

a. Given the Bessel`s differential equation $x^2y'' + xy' + \left(x^2 - n^2\right)y = 0$, about the point x = 0.

i. Obtain the indicial equation of the differential equation (8 marks)

ii. Find the recurrence relation satisfied by coefficients in the series solution of the differential equation and obtain a_2 (5 marks)

b. Obtain a_0 *and* a_n *and* b_n for the Fourier series represented by $f(x)=\begin{cases}2, & -2<x<0\\ x, & 0<x<2\end{cases}$

(7 marks)

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