CHUKA **UNIVERSITY**

UNIVERSITY EXAMINATION

RESIT/SUPPLEMENTARY / SPECIAL EXAMINATIONS
EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE /ARTS, BACHELOR OF SCIENCE, BACHELOR OF ARTS (MATHEMATICS ECONOMICS)

MATH 312/302: REAL ANALYSIS I

STREAMS: **TIME: 2 HOURS**

DAY/DATE: WEDNESDAY 11/08/2021 **2.30 P.M - 4.30 P.M.**

INSTRUCTIONS

- **Answer question ONE and ANY other two questions**
- **Do not write on the question paper**

QUESTION ONE: (30 MARKS)

(a) Define the following terms as used in real analysis

(i). A rational number

(ii). A closed sphere

(iii). A neighborhood of the point x

(iv). A convergence sequence (4 marks)

(b) Given two sets $X\ and\ Y$.

(i) Distinguish the difference of set Y relative to X and the complement of Y relative to X. (2 marks)

(ii) Show that sets $(X\backslash Y)$ and $(X \cap Y)$ are disjoint and that their union is X. (4 marks)

(c) Show that if $xandy$are non-negative real numbers, then $x < y$if andonly if$x^2 < y^2$ (4 marks)

(d) Using the first principle, prove that

(i) $\lim_{n\to\infty}\left(\frac{n}{n+1}\right) = 1$ (3marks)

(ii) $\lim_{x\to\infty} 5^x = \infty$ (3marks)

(iii) $\lim_{x\to -3}(5x+2) = -13$ (3 marks)

(iii)$\lim_{x\to\infty}(x + sinx) = \infty$ (3 marks)

Show that$f(x) = \frac{1}{x}$is continouson the interval $0 < x < 1$ but not uniformly continuous on the same interval. (4 marks)

QUESTION TWO: (20 MARKS)

(a) Find if the following sets are bounded or not and if bounded find the $sups$ and $infs$

(i) $S_1 = \{x \in \mathbb{R}: a \leq x < b\}$ (2 marks)

(ii)$S_2 = \left\{1 + (-1)^n \frac{1}{n}: n \in \mathbf{N}: \right\}$ (2 marks)

(iii) $S_2 = \{1 + (-1)^n . n: n \in \mathbf{N}: \}$ (2 marks)

(b) Find if possible the limit superior and limit inferior of the sequences

(i)$X_n = n(1 + (-1)^n): n \in \mathbf{N}$ (4 marks)

(ii)$X_n = \left(sin\frac{n\pi}{2} + \frac{(-1)^n}{n}\right): n \in \mathbf{N}$ (5marks

(c) Define a countable set. Hence illustrate that the set of rational numbers between $[0, 1]$ is countable whereas the set of real numbers$\mathbb{R}$ is uncountable

(5 marks)

QUESTION THREE: (20 MARKS)

(a) Let S be a non-empty subset of $\mathbb{R}$. Show that the rael number A is the supremum of S if and only if both the following conditions are satisfied.

(i) $x \leq A \;\; \forall x \in S$

(ii) $\forall\, \varepsilon > 0 \;\exists\, x' \in S: A - \varepsilon < \; x' \leq A$ (4 marks)

(b) Consider the set $C[0,1]$ which represents real valued continuous functions on the interval $[0,1]$.Define a function $d: C[0,1] \times C[0,1] \rightarrow R$ by $d(f,g) = \int_0^1 |f(x) - g(x)| dx$.
Show that $(C[0,1], d)$ is a metric space. (7 marks)

(c) (i) Determine whether the limit of the function $f(x) = \sqrt{x}$ exists at $x = 0$

(3 marks)

(ii) Show that if $\lim_{x \to x_0} f(x)$ exists, then that limit is unique (6 marks)

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