CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF

MATH 313/303: REAL ANALYSIS II

STREAMS: Y3S2 **TIME: 2 HOURS**

DAY/DATE: TUESDAY 02/02/2021 **2.30 P.M – 4.30 P.M**

INSTRUCTIONS:

Answer question ALL the questions
Sketch maps and diagrams may be used whenever they help to illustrate your answer
Do not write on the question paper
Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

(a) (i) When is the sequence x_n of elements of real or complex numbers said to be convergent? (2 marks)

(ii) Let x_n, y_n and z_n be sequences of real numbers such that $x_n \leq z_n \leq y_n \quad \forall\, n \geq N$(N is a fixed integer). Let x_n, y_n both converge to the same limit, say $\boldsymbol{l}$. Show that z_n also converges to $\boldsymbol{l}$ $as\ n \to \infty$ (5 marks)

(b) Take $a, b > 0\ (a, b \neq 1)$, prove that $log_a x = \frac{log_b x}{log_b a}$ (3 marks)

© Define and give an example of a periodic function (2 marks)

(d) (i) Define an absolutely convergent series (2 marks)

(ii) Show that in general absolute convergence implies convergence in (K, d) (3 marks)

(e) (i) Let $\sum_{k\in N} x_k$ be a series of real numbers. Prove that if $|x_k| \leq y_k \; \forall \; k \in \boldsymbol{N}$ and $\sum_{k\in N} y_k$ is convergent, then the sum $\sum_{k\in N} x_k$ is also convergent (4 marks)

(ii) Prove that if $p = 1$, then the series $\sum_{n\in N} \frac{1}{n^P}$ is divergent (5 marks)

(f) Define the Fourier series of the function$f(x)$ on the interval $-l \; to \; l$ (4 marks)

QUESTION TWO: (20 MARKS)

(a) (i) Write the general expression of an exponential and logarithmic function whose base is a (2marks)

(ii) By considering $a > 1 \; and \; 0 < a < 1$ for the functions f(x) and g(x) respectively make a comparison of the exponential and logarithmic functions. Hence state any three differences in these graphs. (8marks)

(b) Find the Fourier series of the function defined by

$$f(x) = 0, \quad for - \pi < x < 0, \quad and \qquad f(x) = x \quad for \;\; 0 < x < \pi$$

(10marks)

QUESTION THREE: (20 MARKS)

(a) (i) Describe the Riemann Integrable function f on the interval $[a, b]$ (4 marks)

(ii) Show that a Dirichlet function on the interval $[a, b]$ is not Riemann Integrable. (6 marks)

(b) Show that the function $f(x) = x$ is Riemann Integrable in $[0,1]$ that $\int_0^1 f(x) = \frac{1}{2}$ (10 marks)
