CHUKA UNIVERSITY

UNIVERSITY EXAMINATION

UNIVERSITY EXAMINATION FOR THE AWARD DEGREE OF BACHELOR OF EDUCATION,BSC. GENERAL, ECON/MATH

MATH 304: COMPLEX ANALYSIS I

DAY/DATE : JULY 2021 **TIME: 2HOURS**

INSTRUCTIONS:

Answer Questions ONE (compulsory) and any other TWO Questions

QUESTION ONE (30 MARKS) COMPULSORY

a. Simplify and write the complex expression in the standard form $a+bi$. (1,3,3 Marks)

i. $\frac{1-i}{2}$

ii. $(2-i)^2$

iii. $\left(\frac{1}{2}+\frac{i}{7}\right)\left(\frac{3}{2}-i\right)$

b. If $w=f(z)=\frac{1+z}{1-z}$

i. Determine the point where $f(z)$ is not analytic (2Marks)

ii. Find $\frac{dw}{dz}$ (3Marks)

c. Evaluate the following Limits

i. $\lim_{z\to 1-i}(z^2-5z+10)$ (3Marks)

ii. $\lim_{z\to -2i}\frac{(2z+3)(2-1)}{(z^2-2z+4)}$ (4Marks)

d. Show that is $f(\ )=-2xy+i(x^2-y^2)$analytic (5Marks)

e. Convert the given Complex number into the form indicated

i. $\sqrt{3}-i$ into polar form (3Marks)

ii. $2(Cos120^\circ+iSin120^\circ)$ into Cartesian form (3Marks)

QUESTION TWO (20 MARKS)

a. Solve the following for z

   i. $(2+3i)z=(2-i)z-i$ (3Mark)

   ii. $iz+2i=4$ (4Mark)

b. i. State the Cauchy Integral formula

   ii. Using the Cauchy Integral formula evaluate $\oint_C \frac{z^2-4z+4}{z+i}dz$,where C is the circle $|z|=2$ (5Marks)

c. Compute the Laurent series for the function $f(z)=\frac{z+1}{z^3(z^2+1)}$ on the region $A: 0<|z|<1$ centered at $z=0$. (6Marks)

QUESTION THREE (20 MARKS)

a. Simplify $\frac{14+3i}{2-i}$ and give your answer in the form $x+iy$ (4Marks)

b. Find all the residues of $f(z)=\frac{1}{z^2+2z+10}$ (9Marks)

c. Using DeMoivre's Theorem $(Cos\theta+iSin\theta)^n=Cosn\theta+iSinn\theta$, show that $Tan3\theta=\frac{3Tan\theta-Tan^3\theta}{1-3Tan^2\theta}$ (7Marks)

QUESTION FOUR (20 MARKS)

a. Solve the equation $z^2+4z+5=0$ (3Marks)

b. Evaluate the integral using the residue theorem with $|z|=3$ (13Marks)

c. Find $f(z)=u+iv$, given that $f(z)$ is analytic and $u=x^3-3x^2y$ (4Marks)

QUESTION FIVE (20 MARKS)

a. State DeMoivre's Theorems on:

   i. Powers of complex numbers (2Marks)

   ii. n^{th} roots of complex numbers (2Marks)

b. Using the Theorems stated in 5a above:

i. Expand $z=(1+i)^9$ (4Marks)

ii. Find the square roots of $z=2+i2\sqrt{3}$ (5Marks)

c. Determine the region of the w-plane into which the region bounded $x=1.y=1\ and\ x+y=1$ by is mapped by the transformation $w=z^2$ (7Marks)