CHUKA UNIVERSITY

UNIVERSITY EXAMINATION
RESIT/SUPPLEMENTARY / SPECIAL EXAMINATIONS
EXAMINATION FOR THE AWARD OF BACHELOR OF EDUCATION (SCIENCE/ARTS)

MATH 304/315: COMPLEX ANALYSIS

STREAMS: **TIME: 2 HOURS**

DAY/DATE: WENESDAY 03/11/2021 **11.30 A.M - 1.30 P.M.**

INSTRUCTIONS

- **Answer Question ALL the Questions**

QUESTION ONE (30 MARKS)

a) Express $z = \dfrac{\sqrt{1+x^2}+ix}{x-i\sqrt{1+x^2}}$ in the form $a+ib$, where a and b are real numbers hence Find the complex number z^3. (5 marks)

b) Solve the equation $\cos z = -2$ (5 marks)

c) Evaluate the following complex integrals

i. $\displaystyle\int \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z+4)} dz$ where C is the circle $|z| = 3$ (5 marks)

ii. $\displaystyle\int \frac{e^{2z}}{(2z+1)^4} dz$ where C is the circle $|z-1| = 3$ (5 marks)

d) Find the analytic function $w = f(z)$ from its known real part $u(x, y) = 2e^x \cos y$ (5 marks)

e) Find the Maclaurin's series for the function $f(z) = \sin z$ (5 marks)

QUESTION TWO (20 MARKS)

a) Verify that the function $f(z) = x^3 \sin y - 3xy^2 + i(3x^2 y - y^3 \cos 2x$ is not analytic

(5 marks)

b) Given that $z = re^{i\theta}$, show that $\text{Re}[\log(z-1)] = \frac{1}{2}\log(1 - 2r\cos\theta + r^2)$ for $z \neq 1$.

(5 marks)

c) Without using the cauchy theorem, evaluate $\int_c \overline{zdz}$ from $z = 0$ to $z = 4 + i$ along the curve

$z = 0$ to $z = 2i$ and the line from z= $2i$ to $4 + 2i$ (5 marks)

d) Determine the region of convergence of the series $\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3 4^n}$ (5 marks)

QUESTION THREE (20 MARKS)

a) (i) State and prove the Cauchy's theorem of integration

(ii) Hence show that $\oint_c \frac{e^{2z}}{z^2 - 3} dz = 0$ where C:= $|z + 2i| = 1$ (10 marks)

b) State without proof the residue theorem and use it to evaluate the integral

$\oint_c \frac{e^z}{z^2(z+2)\ (z^2-5)} dz$ C; $|z-2| = 6$ (10 marks)

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