

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION

MATH 315/304: COMPLEX ANALYSIS

STREAMS: BED

TIME: 2 HOURS

DAY/DATE: WEDNESDAY 03/02/2021

8.30 A.M. – 10.30 A.M.

INSTRUCTIONS: ANSWER QUESTION ALL THE QUESTIONS

QUESTION ONE (30 MARKS)

- a) Determine the three distinct roots of the complex number $z = 2 + 2i$ (4 marks)
- b) Evaluate the limits of the following functions
- i. $\lim_{z \rightarrow \pi} \frac{\sin z}{\pi - z}$ (2 marks)
- ii. $\lim_{z \rightarrow i} \frac{e^{\pi z} + 1}{z^2 + 1}$ (2 marks)
- c) Solve the equation $e^{2z} = -1 + i\sqrt{3}$ (4 marks)
- d) Determine the region of convergence of the series $\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3 4^n}$ (5 marks)
- e) Evaluate the following complex integrals
- i. $\int \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z+4)} dz$ where C is the circle $|z+4| = 2$ (4 marks)
- ii. $\int \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z-1| = 3$ (4 marks)
- f) Find the Maclaurin's series for the function $f(z) = \cos z$ (5 marks)

QUESTION TWO (20 MARKS)

- a) Show that the function $u(x, y) = 2e^{-x} \sin y$ is harmonic and find the analytic function $w = f(z)$ from its known real part $u(x, y) = 2e^{-x} \sin y$ (10 marks)
- b) Use the residue theorem to evaluate the integral $\oint_c \frac{z^2 + 5}{z^2(z+1)^3(z^2+1)} dz$ where $c; |z-2| = \frac{5}{2}$ (10 marks)

QUESTION THREE (20 MARKS)

- a) Show that $\cos^{-1} z = -i \log[z + i(1-z^2)^{\frac{1}{2}}]$ (5 marks)
- b) Verify that the function $f(z) = x^3 \sin y - 3xy^2 + i(3x^2y - y^3 \cos 2x)$ is not analytic (5 marks)
- c) By expanding the function $f(z) = \frac{1}{1+z^2}$ about $z = 0$ and $z = i$, explain the difference between the two expansions (10 marks)
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