

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF BACHELOR OF SCIENCE

MATH 304: COMPLEX ANALYSIS

STREAMS: BSC (AGED) Y3S2

TIME: 2 HOURS

DAY/DATE: THURSDAY 12/8/2021

8.30 A.M. – 10.30 A.M

INSTRUCTIONS: ANSWER ALL QUESTIONS

QUESTION ONE (30 MARKS)

- a) Determine the three distinct roots of the complex number  $z = 2 + 2i$  (4 marks)
- b) Evaluate the limits of the following functions
- i.  $\lim_{z \rightarrow \pi} \frac{\sin z}{\pi - z}$  (2 marks)
- ii.  $\lim_{z \rightarrow i} \frac{e^{\pi z} + 1}{z^2 + 1}$  (2 marks)
- c) Solve the equation  $e^{2z} = -1 + i\sqrt{3}$  (4 marks)
- d) Determine the region of convergence of the series  $\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3 4^n}$  (5 marks)
- e) Evaluate the following complex integrals
- i.  $\int \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z+4)} dz$  where C is the circle  $|z+4|=2$  (4 marks)
- ii.  $\int \frac{e^{2z}}{(z+1)^4} dz$  where C is the circle  $|z-1|=3$  (4 marks)
- f) Find the Maclaurin's series for the function  $f(z) = \cos z$  (5 marks)

**QUESTION TWO (20 MARKS)**

- a) Show that the function  $u(x, y) = 2e^{-x} \sin y$  is harmonic and find the analytic function  $w = f(z)$  from its known real part  $u(x, y) = 2e^{-x} \sin y$  (10 marks)
- b) Use the residue theorem to evaluate the integral  $\oint_c \frac{z^2 + 5}{z^2(z+1)^3(z^2+1)} dz$  where  $c; |z-2| = \frac{5}{2}$  (10 marks)

**QUESTION THREE (20 MARKS)**

- a) Show that  $\cos^{-1} z = -i \log[ z + i(1 - z^2)^{\frac{1}{2}} ]$  (5 marks)
- b) Verify that the function  $f(z) = x^3 \sin y - 3xy^2 + i(3x^2y - y^3 \cos 2x)$  is not analytic (5 marks)
- c) By expanding the function  $f(z) = \frac{1}{1+z^2}$  about  $z = 0$  and  $z = i$ , explain the difference between the two expansions (10 marks)
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