CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THEAWARD OF DEGREE OF BACHELOR OF EDUCATION SCIENCE, BACHELOR OF EDUCATION ARTS, BACHELOR OF ECONOMICS AND MATHEMATICS AND BACHELOR OF SCIENCE

MATH 302: REAL ANALYS I

STREAMS: AS ABOVE **TIME: 2 HOURS**

DAY/DATE: MONDAY 29/03/2021 **8.30 A.M. – 10.30 A.M.**

INSTRUCTIONS:

- Answer Question One and ANY two questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE (30 MARKS)

a) (i) Prove that despite the fact that there exists other numbers on the real number line which are not rational, between any two rational numbers there is another rational number. (2 marks)

(ii) Show that $\forall x, y \in \mathbb{R},\ x \geq 0\ and\ y \geq 0, x < y\ iff\ x^2 < y^2$ (3 marks)

b) For any two sets X and Y, show that $(X \cap Y)^c = X^c U Y^c$ (3 marks)

c) Given that $a\ and\ b$ are rational numbers with $b \neq 0$ and α is irrational number such that $a - \alpha b = \beta$, show that β is an irrational number and hence show that $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ is an irrational number. (5 marks)

d) Use the comparison test to determine the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)2^{2n-1}}$

(3 marks)

e) Consider the set $= \{\frac{n+1}{n} : n \in \mathbb{N}\}$. Show that the $Sup\ S = 2$. (3 marks)

f) Use the first principle to show that $\lim_{n\to\infty} \left|1 + (-1)^n \frac{1}{n^2}\right| = 1$. (3 marks)

g) Let $\lim_{n\to\infty}\{x_n\} = t$ and $\lim_{n\to\infty}\{y_n\} = m$ where $\{x_n\}$ and $\{y_n\}$ are sequences of real numbers. Then show that the $\lim_{n\to\infty}\{x_n y_n\} = tm$

(5 marks)

h) Show that a function which is uniformly continuous on an interval is continuous on that interval. (3 marks)

QUESTION TWO (20 MARKS)

a) Show that $f(x) = x^2$ is uniformly continuous on the interval $[0,2]$ but not uniformly continuous on the interval $(0, \infty)$. (5 marks)

b) Show that a function $F(x) = \frac{1}{x}$ is not continuous on the interval $(0,1)$ (4 marks)

c) Given $f(x) = x^2 - 5x$ show that the $\lim_{x\to 2} f(x) = -6$ and hence determine a value for $\delta > 0$ associated with $\varepsilon > 0$ in accordance with the definition of a limit of a function. (6 marks)

d) Show that the sequence $\left\{\frac{1}{2^n}\right\}$ is a Cauchy sequence. (5 marks)

QUESTION THREE (20 MARKS)

a) Let $2x_n\}$ be a sequence of real numbers, prove that if $\{x_n\}$ converges, then its limit is unique.

(6 marks)

b) Prove that a sequence $\{y_n\}$ of real numbers converges to a limit y if and only if every subsequence of $\{y_n\}$ converges to y. (5 marks)

c) Prove that every convergent sequence is bounded and with the help of an example show that the converse is not necessarily true. (9 marks

QUESTION FOUR (20 MARKS)

a) Use the Cauchy's integral test to show that $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges if $p > 1$and diverges if $p \leq 1$ (6 marks)

b) Prove that set of real numbers $\mathbb{R}$ is uncountable. (5 marks)

c) Suppose that $\sum_{k=1}^{\infty} x_k$ and $\sum_{k=1}^{\infty} y_k$ are positive series if $x_k \leq y_k \ \ \forall\ k = 1,2,3, \ldots$ show that $\sum_{k=1}^{\infty} x_k$ will converge if $\sum_{k=1}^{\infty} y_k$ converges. (4 marks)

d) Let M and N be neighbourhood of point x. Show that $M \cap N$ is also a neighbourhood of x. (5 marks)

QUESTION FIVE (20 MARKS)

a) Consider three sets X$, Y, Z$ show that $Z|(XUY) = (Z|X) \cap (Z|Y)$ (3 marks)

b) Show that the sets $(B|A)$ and $(A \cap B$ are disjoint and their union is B. (6 marks)

c) Prove that in any space, the finite intersection of a family of open sets is open. (4 mark$s$)

d) Use the Intermediate Value Theorem to show that the equation $e^{2-3x} - e^{-x}$ has a real root between 0 and 3. (7 marks)
