

**SECOND YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE**

MATH 244 – : INTRODUCTION TO STATISTICAL INFERENCE

DURATION: 2 HOURS

DATE:

TIME:

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE

a) Define and differentiate the meaning of the below terms

i) Descriptive statistics and Inferential statistics (2 marks)

ii) Population and Sample (2 marks)

iii) Simple random sample and Systematic random sample (2 marks)

iv) Point estimate and interval estimate (2 marks)

b) A tire manufacturer wishes to investigate the tread life of its tires. A sample of 10 tires driven 50,000 miles revealed a sample mean of 0.32 inches of tread remaining with a standard deviation of 0.09 inches. Construct a 95 percent confidence interval for the population mean. Would it be reasonable for the manufacturer to conclude that after 50,000 miles the population mean amount of tread remaining is 0.30 inches? (5 marks)

c) The total number of new motor insurance claims reported to a particular branch of an insurance company on successive days during a randomly selected month can be considered to come from a Poisson distribution with $\lambda = 5$. What are the mean and variance of a sample mean based on 30 days figures? (3 marks)

d) Sample variance is given by

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

Prove that $E[s^2] = \sigma^2$ (6 marks)

e) In the claims department of Madison insurance office various quantities are computed at the end of each day's business. On Monday, 10 claims are received for a particular class of policy. The mean claim amount is calculated to be Kshs 426 and the standard deviation to be Kshs 112.

On Tuesday it is found that one of Monday's claims for Kshs.545, was classified wrongly and it is removed from the set of 10 claims. Calculate the resulting mean and standard deviation of the reduced set of 9 claims. (8 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO

a) Differentiate between mean and median (3 marks)

b) A random sample from a **Bin (n, p)** distribution yields the following values:

4, 2, 7, 4, 1, 4, 5, 4

Calculate method of moments estimates of **n** and **p**.(8 marks)

c) The average IQ of a sample of 50 Chuka university students was found to be 132. Calculate a symmetrical 95% confidence interval for the average IQ of the university students, assuming that IQs are normally distributed. It is known from previous studies that the standard deviation of IQs among students is approximately 20. (5 marks)

d) Outline the roles of normal distribution in statistics.(4 marks)

QUESTION THREE

a) Calculate the spearman’s rank correlation coefficient for the below data.(5marks)

x	10	8	12	15	8	10
y	7	4	6	7	9	8

b) Determine (2 marks)

$$P(F_{3,9} < 3.863)$$

c) If $X \sim N(14, 20)$ calculate:

i) r such that $P(X > r) = 0.41294$ (3 marks)

ii) $P(X > 20)$ (2 marks)

d) Company ABC has developed a new line of products. Top management is attempting to decide on the appropriate marketing and production strategy. 3 strategies are being considered: A (Aggressive), B (basic) and C (Cautious). The market conditions under study are denoted by S (Strong) or W (Weak). Management's best estimate of the net profits (in \$, mil) in each case is given in the following payoff table.

Decision	State of nature	
	S	W
A	30	-8
B	20	7
C	15	10

Management's estimates of the probabilities of a strong or a weak market are 0.45 and 0.55 respectively.

- i) Draw a decision tree to represent this problem (6 marks)
- ii) Which alternative is the best choice? And Why? (3 marks)

QUESTION FOUR

- a) Discuss the three elements of a decision (6 marks)
- b) Suppose that we have reason to believe that the readings x_1, x_2, \dots, x_{16} obtained from an experiment were a random sample from a $N(\mu, 4)$ distribution, and we wish to test $H_0 : \mu = \mu_0 = 36.0$ versus $H_1 : \mu < 36.0$. If the observed value of \bar{x} is 34.4, what would be the outcome of the test at the 5% significance level? (4 marks)
- c) Suppose $X \sim N(\mu, \sigma^2)$ with σ^2 unknown and let 38.8, 39.2, 39.4, 39.0, 38.6 be a random sample of observations on X . Test at the 5% level whether $\mu = 40$ or not. What is the conclusion for a test at the 1% significance level? What about the 0.1% level? Interpret your findings. (10 marks)

QUESTION FIVE

- a) Define what is a nonparametric tests (2 marks)
- b) List 3 advantages and 3 disadvantages of nonparametric test (6 marks)
- c) Calculate the:

$$n = 100 \quad \sum (x_i - \bar{x})^2 = 856,934.91 \quad \sum (x_i - \bar{x})^3 = -11,949,848.3946$$

- i) skewness (2 marks)
- ii) coefficient of skewness. (3 marks)
- d) With the introduction of HDTV, TV stations will eventually have to start producing and broadcasting their shows in this new (digital) standard. NBC, trying to decide on their own strategy, conducted a poll in major metropolitan areas and found that 39 out of 200 households was likely to purchase an HDTV within the next 18 months.

Give an estimate and a 90% confidence interval for the true proportion of American households in major metropolitan areas that are likely to purchase an HDTV over the next 18 month period. (7 marks)