**CHUKA UNIVERSITY**

**UNIVERSITY EXAMINATIONS**

**SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE, ARTS AND EDUCATION, BACHELOR OF PHYSICS, BACHELOR OF INDUSTRIAL CHEM, BACHELOR OF SCIENCE (COMPUTER SCIENCE, MATHEMATICS, ACTURIAL SCIENCE)**

**MATH 242: PROBABILITY AND STATISTICS II**

**STREAMS: BSC (ECON &STAT, PHYS, COMP SCI, MATHS, ACTURIAL SCI, INDUCTRIAL SCI)** **TIME: 2 HOURS**

**DAY/DATE: THURSDAY 08/07/2021** **8.30 A.M – 10.30 A.M**

**INSTRUCTIONS:**

**Answer question one and any other two questions**

**QUESTION ONE (30 MARKS)**

(a) Suppose that X and Y be two continuous random variables with joint density function.

f(x,y) =f (x) = $\begin{cases} kx^3y^3, & 0 \le x \le 2,\ 0 \le y \le 2 \\ 0 & otherwise \end{cases}$

(i) Find the value of k [2 marks]

(ii) Determine whether variables X and Y are independent. [3 marks]

(iii) Find = P ( X ≤ ½ : $Y > 1$) [2 marks]

(b) Let X and Y have the joint density function.

f(x,y) = $\begin{cases} 4\,xy, & 0 \le x \le k, 0 \le y \le 1 \\ 0, & otherwise \end{cases}$

Find

(i) Show that the value of k is 1 [3 marks]

(ii) The conditional PDF of Y given X = x [3 marks]

(c) Suppose the joint probability distribution function of X and Y is represented by the following table below.

| | Y | | | |
|---|---|---|---|---|
| X | 1 | 2 | 3 | 4 |
| 0 | 0.059 | 0.1 | 0.05 | 0.001 |
| 1 | 0.093 | 0.12 | 0.082 | 0.003 |
| 2 | 0.065 | 0.102 | 0.1 | 0.01 |
| 3 | 0.050 | 0.075 | 0.07 | 0.02 |

(i) Find the marginal probability distributions of X and Y [4 marks]

(ii) Find E(Y/X=3) and var (Y/X =3) [8 marks]

(d) A fair coin is tossed 100 times. Show that the probability that the number of heads will be between 30 -70 is at least 0.94. [5 marks]

**QUESTION TWO (20 MARKS)**

(a) The joint moment generating function of f(x,y) is given as

$$M(t_{1,}t_2) = \left[\frac{2}{3}e^{t_1} + \frac{1}{3}e^{t_2}\right]$$

Find ;

i. Marginal moment generating function of X [1 mark]

ii. Cov(X,Y) and Var(X) [8 marks]

(b) Suppose that X and Y are bivariate normal with E(X) =1, E(Y) =2, Var (X) = var (Y) = $\frac{1}{3}$ and the correlation of $\frac{1}{2}$. Calculate P(2.2$< y < 3.2/X = 3$) [4 marks]

(c) Two normal ransom variables X and Y have joint p.d.f given by

$$f(x) = k\ exp\left(-\frac{25}{18}\right)\left\{\left(\frac{x-20}{4}\right)^2 - 0.08\ (x-20)(y-30) + \left(\frac{y-30}{5}\right)^2\right\}$$

Where k is a constant

(i) Determine the value of the correlation coefficient [2 marks]

(ii) Find P(34$< Y < 37/X = 25$) [5 marks]

## QUESTION THREE

(a) Let $y_1 < y_2 < y_3 < y_4$ denote order statistics of a random sample of size 4 from a population with pdf given by;

$$F(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0 & elsewhere \end{cases}$$

Determine

(i) The p.d.f of $y_2$ [4 marks]

(ii)The P[$y_2 < \frac{1}{3}$] [2 marks]

(iii) E ($y_2$ ) [2 marks]

(b) Suppose that $x_1$ and $x_2$ are independent random variables and that the p.d.f of each of these variables is;

$$F(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & elsewhere \end{cases}$$

Find the p.d.f of $y_1 = x_1 + 2x_2$ [8 marks]

(c) Let X have the binomial p.m.f

$$P(x = x) = \begin{cases} \frac{3!}{x!(3-x)!}(2/3)^x(1/3)^{3-x}, x = 0,1,2,3 \\ 0 \qquad elsewhere \end{cases}$$

Find the p.m.f of Y = $x^2$ [4 marks]

## QUESTION FOUR (20 MARKS)

Given that $x_1$ and $x_2$ are jointly normally distributed random variables and that $\sigma_1^2 = \sigma_2^2 = 1$ and $\mu_1 = \mu_2 = 0$. Find the joint p.d.f of $y_1 = 2x_1 + x_2$ and $y_2 = x_1 - x_2$ [10 marks]

(b) The joint p.d.f of x and y is given by.

$$F(x\ y) = \begin{cases} 8xy & 0 < y < x < 1 \\ 0 & elsewhere \end{cases}$$

Find

(i) f(x) [2 marks]

(ii) f(y)/x) [2 marks]

(iii) E (y/x) [3 marks]

(iv) Var (y/x) [3 marks]

**QUESTION FIVE (20 MARKS)**

Let $x_1$ and $x_2$ be two independent R.V.S having a poisson distribution with parameters $\lambda_1$ and $\lambda_{2.}$
Find the probability distribution function of Y = $x_1 + x_2$. [20 marks]

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