CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE (PHSICS, COMPUTER SCIENCE, APPLIED COMPUTER SCIENCE, ECONOMICS AND STATISTICA, MATHEMATICS, INDUSTRIAL CHEMISTRY, ELECTRICAL AND ELECTRONICS ENGINEERING, ACTURIAL SCIENCE), BACHELOR OF EDUCATION (SCIENCE, ARTS), BACHELOR OF ARTS (ECONOMICS AND MATHEMATICS, ECONOMICS AND SOCIOLOGY) (MAIN CAMPUS - REGULAR BASED)

MATH 241: PROBABILITY AND STATISTICS I

STREAMS: AS ABOVE

TIME: 2HRS

INSTRUCTIONS:

- **Answer question one and any other two questions**
- **All workings must be shown clearly**

Question One: 30 marks

a) Explain the difference between the following terms:

i). Discrete random variables and continuous random variables (2mks)

ii). Probability mass function and cumulative distribution function (2mks)

iii). Expectations and variance of continuous random variables (2mks)

b) Outline properties of probability distribution function (2mks)

c) Suppose a fair coin is tossed twice. Let X be the number of tails that can come up. Obtain the probability mass function of the random variable X. (3mks)

d) Find the constant C such that the function $f(x) = \begin{cases} CX^2 & 0 < x < 3 \\ 0 & otherwise \end{cases}$ is a probability density function. Hence find $p(1 < x < 2)$ (4mks)

e) The density function of a random variable X is given by $f(x) = \begin{cases} \frac{1}{2}x & 0 < x < 2 \\ 0 & otherwise \end{cases}$

Determine the

i). Mean X (3mks)

ii). Variance of X (3mks)

f) Find the area under the standard normal curve between $z = -0.46$ and $z = 2.21$

(2mks)

g) A multiple- choice test contains 25 questions each 4 answers. Assume a student guesses on each question.

i) What is the probability that the student answer more than 20 questions correctly.

(3 mks)

i) Less than five questions correctly.

(3 mks)

Question Two: (20 marks)

a) Given the Beta density function as;

$$f(x) = \begin{cases} \frac{1}{B(a,b)} X^{a-1}(1-X)^{b-1} & 0 \le x \le 1 \\ 0 \; otherwise \end{cases}$$

i. Determine the mean of X (3mks)

ii. Obtain the variance of X (5mks)

b) Given the random variable X with Poisson distribution function as

$$f(x) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!} & x = 0,1,2,\ldots \\ 0 & otherwise \end{cases}$$

i. Find $E(X)$ (4mks)

ii. Find $Var(X)$ (4mks)

iii. If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001, determine the probability that out of 2000 individuals exactly 3 individuals will suffer a bad reaction (4mks)

Question Three: (20 marks)

a) i). Outline properties of moment generating function (2mks) ii). (6mks)

c) Suppose that $X \sim N(\mu, \delta^2)$, find the

i). Moment generating function $M_x(t)$ (4mks)

ii). Mean $E(x)$ (4mks)

iii). Variance $var(x)$ (4mks)

Question Four: (20marks)

a) Suppose we toss a fair coin 20 times. What is the probability of getting between 9 and 11 heads? (4mks)

b) Let X be a discrete random variable with a geometric distribution given as

$$f(x) = \begin{cases} q^x p & x = 0,1,2 \ldots \\ 0 & otherwise \end{cases}$$

Find

i). Moment generating function $M_{x(t)}$ (3mks)

ii) Mean E(X) (3mks)

c) Suppose X has a Poisson distribution with expected value λ given by

$$P(x{:}\lambda) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots. \\ 0, otherwise \end{cases}$$

Find its factorial moment generating function 6mks

d) Given a random variable X whose probability distribution function is given as

$$f(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 0,1, . . n \\ 0 & otherwise \end{cases}$$

i). Find the mean of X (3mks)

ii). Determine the variance of x (4mks)

Question Five: (20 marks)

a) i) Suppose a random variable x has the m.g.f. $M_x(t) = e^{3t+4t^2}$. Determine the mean and variance of x. Hence or otherwise give its p.d.f.

(4 mks)

ii) Use the gamma function to show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

(4 mks)

ii) Given the moment generating function of Gamma distribution as $M_x(t) = (1 - \beta t)^{-\propto}$ use it to determine the mean and variance of Gamma distribution

(6 mks)

b) Let X be a discrete random variable with the shifted geometric distribution with parameter p. That is;

$$P(X = x) = \begin{cases} pq^{x-1}, x = 1, 2, ... \\ 0, elsewhere \end{cases}$$

Find the probability generating function of X(6 mks)