CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS 2021

FIRST YEAR EXAMINATIONS FOR THE AWARD OF BACHELOR OF SCIENCE IN MATHEMATICS.

MATH 223: OPTIMIZATION TIME: 2 HOURS

INSTRUCTIONS

Answer question one and any other two questions

Adhere to the instructions on the answer booklet.

QUESTION ONE Compulsory.

a. Define the following terms

i. Operations Research 2mks

ii. A model in the sense used in operations research 2mks

b. A company manufactures two products *X* and *Y*, by using the three machines *A*, *B*, and *C*. Each unit of *X* takes 1 hour on machine A, 3 hours on machine B and 10 hours on machine *C*. Similarly, product *Y* takes one hour, 8 hours and 7 hours on Machine *A*, *B*, and *C* respectively. In the coming planning period, 40 hours of machine *A*, 240 hours of machine *B* and 350 hours of machine *C* is available for production. Each unit of *X* brings a profit of ksh 5/- and *Y* brings ksh. 7 per unit. Obtain an equation for the objective function and the constraints for maximizing the profit. 5mks

c. Maximize $f(x, y) = \mathbf{143}x + \mathbf{60}y$ subject to the constraints: 6mks

$$\begin{aligned} x + y &\le 100 \\ 120x + 210y &\le 15000 \\ 110x + 30y &\le 4000 \\ x, y &\ge 0 \end{aligned}$$

d. Maximize $g = -f = -3x - 2y$ subject to 5mks

$$\begin{aligned} x + y &\ge 10 \\ x - y &\le 15 \end{aligned}$$

e. A patient consult a doctor to check up his ill health. Doctor examines him and advises him that he is having deficiency of two vitamins, vitamin *A* and vitamin *D*. Doctor advises him to consume vitamin *A* and *D* regularly for a period of time so that he can regain his health. Doctor prescribes tonic *X* and tonic *Y*, which are having vitamin *A*, and *D* in certain proportion. Also advises the patient to consume **at least** 40 units of vitamin *A* and 50 units of vitamin Daily. The cost of tonics *X* and *Y* and the proportion of vitamin *A* and *D* that present in *X* and *Y* are given in the table below.

Vitamins	*Tonics*		*Daily requirement in units.*
	X	*Y*	
A	2	4	40
D	3	2	50
Cost in Rs. per unit.	5	3	

Formulate a LP. problem to minimize the cost of tonics. 5mks

f. State five steps to solving Operations Research problems. 5mks

QUESTION TWO

a. State 5 types of optimization problems 5mks

b. A farmer has 80 hectares of his farm available for planting maize and cabbages. He must grow at least 10 hectares of maize and 20 hectares of cabbages to meet demands. He prefers to plant more cabbages than maize but his work force and equipment will only allow him to cultivate a maximum of three times the quantity of cabbages to that of maize.

i. Represent the information above as a system of inequalities. 3mks

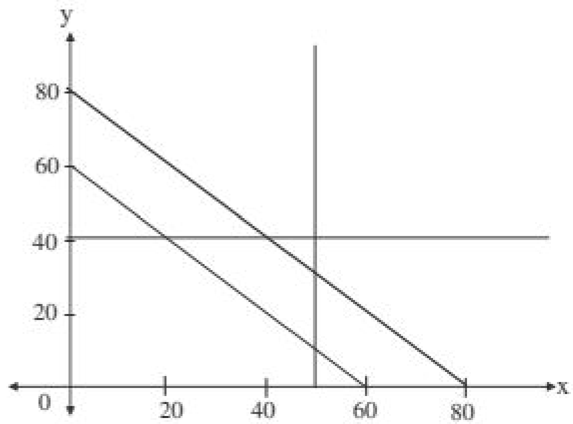
ii. Sketch a graph of these inequalities. 4mks

iii. If the profit on maize is ksh 800 per ha and on cabbages ksh 500 per ha, how should the farmer plant the two crops to make a maximum profit and what is this profit 5mks

c. State three types of models classified according to Structure 3mks

QUESTION THREE

a. A factory makes two types of beds, type A and type B beds. Each month x number of type A beds and y of type B beds are produced. The following constraints control monthly production: Not more than 50 beds of type A and 40 beds of type B can be made. To show a profit at least 60 beds in all must be made. The maximum number of beds that can be produced is 80. The diagram shows the four constraints. Write down in terms of x and y the inequalities that represent these constraints. 4mks



ii. If the objective function is given by the equation $y = -2x + \frac{P}{150}$, where P is the monthly profit in Ksh, Find the profit per bed of the two types of bed. 3mks

iii. How many of each type of bed must be produced per month to maximise profit and What is the maximum profit? 4mks

iv. Explain how the production would be affected if the objective function was $y = -x + \frac{P}{150}$ 4mks

b. A company manufactures two products *A* and B, which require resources. The resources are the machines *M*1, *M*2, and *M*3. The available capacities for the machines are 50,25,and 15 hours respectively per week in the planning period. Product A requires 1 hour of machine *M*2 and 1hour of machine *M*3. Product *Y* requires 2 hours of machine *M*1, 2 hours of machine *M*2 and 1 hour of machine *M*3. The profit contribution of products *X* and *Y* are ksh .5/- and ksh.4/- respectively. Obtain an equation for the objective function and the constraints for maximizing the profit. 5mks

QUESTION FOUR

a. A retail store stocks two types of shirts *A* and *B*. These are packed in attractive cardboard boxes. During a week the store can sell a maximum of 400 shirts of type *A* and a maximum of 300 shirts of type *B*. The storage capacity, however, is limited to a maximum of 600 of both types combined. Type *A* shirt fetches a profit of . 2/- per unit and type *B* a profit of 5/- per unit. How many of each type the store should stock per week to maximize the total profit? Formulate a mathematical model of the problem. 6mks

b. PARLOK Ltd has two products Heaven and Hell. To produce one unit of Heaven, 2 units of material X and 4 units of material Y are required. To produce one unit of Hell, 3 units of material X and 2 units of material Y are required. Only 16 units of material X and 16 units of material Y are available. Material X cost ksh. 2.50 per unit and Material Y cost ksh. 0.25 per unit respectively. Formulate an LP Model and solve it graphically.

6mks

c. Maximize $y = 0.05x + 0.09y + 0.08z$, subject to the following constraints:

$$x + y + z \leq 150$$
$$x \leq 75$$
$$y \leq 75$$
$$z \leq 75$$

8mks.

QUESTION FIVE

a. Maximize: f = -2y + 5z subject to the following constraints: 6mks

$$3x - 2y + z \leq 8$$
$$-4x + 3y - z \leq 4$$
$$2x - 3y - 6z \leq 6$$
$$x - y + z \geq 1$$
$$x + y + z = 5$$

b. Minimize: f = 32x +12y subject to the following constraints: 6mks

$$4x + 3y \geq 6$$
$$8x + 2y \geq 5$$

with x and y nonnegative.

10 grams of Alloy A contains 2 grams of copper, 1 gram of zinc and 1 gram of lead. 10 grams of Alloy B contains 1 gram of copper, 1 gram of zinc and 1 gram of lead. It is required to produce a mixture of these alloys, which contains at least 10 grams of copper, 8 grams of zinc, and 12 grams of lead. Alloy B costs 1.5 times as much per Kg as alloy A. Find the amounts of alloys *A* and *B*, which must be mixed in order to satisfy these conditions in the cheapest way.