CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF EDUCATION SCIENCE /ARTS; BSc. MATHEMATICS; ECONOMICS AND STATISTICS; BACHELORS OF ARTS(MATHS-ECONS)

MATH 222: VECTOR ANALYSIS

STREAMS: "as above" **TIME: 2HRS**

DAY/DATE: ……………………. **………………………..**

INSTRUCTIONS:

- Answer question **ONE** and **TWO** other questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

a) Prove that if $\vec{a}$ and $\vec{b}$ are non-collinear vectors and $x_1\vec{a} + y_1\vec{b} = x_2\vec{a} + y_2\vec{b}$, then $x_1 = x_2$ and$y_1 = y_2$ (2marks)

b) Find the directional derivative of $\phi = x^2 y + xz$ in the direction of the vector $2i - 2j + k$ at (1,-2,2) (4marks)

c) The position vector of a particle in space is given by the equation $\vec{s} = 2\cos 2t\,\hat{\imath} + 2\sin 2t\,\hat{\jmath} + 3t\hat{k}$

(i) Find the equations of the velocity and acceleration at any time t (2 marks)

(ii) Find the initial speed of the particle (1 mark)

d) Find the work done in moving a particle once around a circle C in the x-y plane if the circle has centre at the origin and radius 3 units and the force field is given by

$\overrightarrow{F} = (2x - y + z)\hat{i} + (x + y - z^3)\hat{j} + (x - 6y + z)\hat{k}$ (4 marks)

e) Determine the unit tangent vector at the point where $t = 2$ on the curve $x = t^2 + 1,\ y = 4t - 3,\ z = 2t^2 - 6t$ (3marks)

f) Find the volume of the region to the intersecting cylinders $x^2 + y^2 = 16$ and $z^2 + y^2 = 16$ (5 marks)

g) Find an equation for the plane determined by the points P1(2, -1, 1), P2(-3, -2, 1) and P3(,-1, 3, 2) (4 marks)

h) Given that $\emptyset = 45x^2 y$, evaluate its volume integral such that the volume space is bounded by planes $4x + 2y + z = 8,\ x = 0, y = 0, z = 0.$ (5 marks)

QUESTION TWO: (20 MARKS)

a) Given that A = 3i +j +2k and B = i - 2j-4k are the position vectors of points P and Q respectively. Find an equation for the plane passing through Q and perpendicular to line PQ. (5 marks

b) Prove that $div(curl(\overrightarrow{A})) = 0$ (5 marks)

c) Given the space curve $r(t) = \langle t, 3\sin t, 3\cos t \rangle$, find:

i. The tangent vector $\hat{T}$ (2 marks)

ii. The principal normal $\hat{N}$ and curvature κ (2 marks)

iii. The unit binormal $\hat{B}$ (2 marks)

d) show that $\nabla^2\left(\frac{1}{r}\right) = 0$ (4 marks)

QUESTION THREE: (20 MARKS)

a) Given the vector function $\overrightarrow{F} = (y^2 \cos x + z^3)i + (2y\sin x - 4)j + (3xz^2 + 2)k$

i. Show that a conservative force field (2 marks)

ii. Find the scalar potential (3 marks)

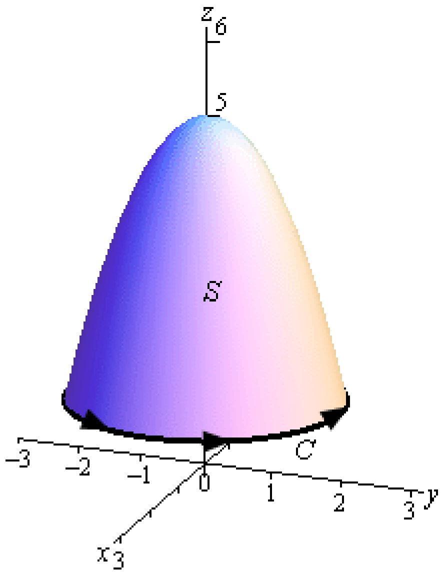
iii. Evaluate $\int_c \overrightarrow{F}.\overrightarrow{dr}$ where c is a straight line from $(0,-1,1)$ to $(\frac{\pi}{2},-1,2)$ (2 marks)

b) State and Verify divergence theorem for $\overrightarrow{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 4, 0 \le z \le 3$ (13 marks)

QUESTION FOUR: (20 MARKS)

a) Evaluate the line integral $\int_c y^3 dx - x^3 dy$ where C is the positively oriented circle of radius 2 centered at the origin (10 marks)

b) Use Stoke's theorem to evaluate $\iint_S curl\,\vec{F}\,\vec{ds}$ where $\vec{F} = z^2 i - 3xyj + x^3y^3k$ and S is the part of the surface $z = 5 - x^2 - y^2$ above the plane $z = 1$. Assume S is oriented upwards



(10 marks)

QUESTION FIVE: (20 MARKS)

a) Find the volume of the parallelepiped with adjacent sides

$\vec{u} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$, $\vec{v} = -\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$, $\vec{w} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ (4 marks)

b) If $\vec{A} = x^2yz\hat{\imath} - 2xz^3\hat{\jmath} + xz^2\hat{k}$ and $\vec{B} = 2z\hat{\imath} + y\hat{\jmath} - x^2\hat{k}$, find $\frac{\partial^2}{\partial_x \partial_y}\overrightarrow{(\vec{A}\,X\vec{B})}$ at (1,0,-2)

(4 marks)

c) Verify Green's theorem for $\int_c x^3dx + 2x(1 + xy)dy$, where C is the unit circle

(12 marks)

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