CHUKA **UNIVERSITY**

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION (SCIENCE,ARTS), BACHELOR OF SCIENCE, BACHELOR OF MATHEMATICS, BACHELOR OF ARTS (MATHS-ECON), BACHELOR OF SCIENCE (ECON STATS)

MATH 222: VECTOR ANALYSIS

STREAMS: "As above' **TIME: 2 HOURS**

DAY/DATE: TUESDAY 23/03/2021 **11.30 A.M – 1.30 P.M**

INSTRUCTIONS:

Answer question **ONE** and **TWO** other questions

- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

(a) Prove that if $\vec{a}$ and $\vec{b}$ are non-collinear vectors and $x_1\vec{a} + y_1\vec{b} = x_2\vec{a} + y_2\vec{b}$, then $x_1 = x_2$ and $y_1 = y_2$ (3 marks)

(b) The position vector of a particle in space is given by the equation $\vec{s} = 2\cos 2t\,\hat{\imath} + 2\sin 2t\,\hat{\jmath} + 3t\hat{k}$

(i) Find the equations of the velocity and acceleration at any time t (2 marks)

(ii) Find the initial speed of the particle (2 marks)

(c) Find the directional derivative of $\emptyset = x^2y + xz$ in the direction of the vectors $2\hat{\imath} - 2\hat{\jmath} + \hat{k}$ at $(1, -2, 2)$ (4 marks)

(d) Show that $\nabla^2\left(\frac{1}{r}\right) = 0$ (4 marks)

(e) Find the work done in moving a particle once around a circle C in the x-y plane if the circle has centre at the origin and radius 3 units and the force field is given by

$\vec{F} = (2x - y + z)\hat{i} + (x + y - z^3)\hat{j} + (x - 6y + z)\hat{k}$ (4 marks)

(f) Find an equation for the plane determined by the points P1(2,-1,1), P2(-3,-2,1) and P3(-1,3,2) (4 marks)

(g) Given that $\emptyset = 45x^2y$, evaluate its volume integral such that the volume space is bounded by planes $4x + 2y + z = 8,\ x = 0, y = 0, z = 0.$ (5 marks)

(h) If $\vec{A} = (2x^2y - x^4)\hat{\imath} + (e^{xy} - ysinx)\hat{\jmath} + (x^2cosy)\hat{k}$, find $\frac{\delta^2 A}{\delta x \delta y}$ (2 marks)

QUESTION TWO: (20 MARKS)

(a) Given that $\vec{A} = 3\hat{\imath} + \hat{\jmath}\ + 2\hat{k}$ and $\vec{B} = \hat{\imath} - 2\hat{\jmath} - 4\hat{k}$ are position vectors of points P and Q respectively. Find an equation for the plane passing through Q and perpendicular to the line PQ. (6 marks)

(b) Given the space curve defined by $\vec{r}(t) = \langle t, 3sint, 3cost\rangle$, Find

(i) The tangent vector $\vec{\mathrm{T}}$ (3 marks)

(ii) The principal normal $\vec{\mathrm{N}}$ and curvature κ (3 marks)

(iii) The Binormal $\vec{\mathrm{B}}$ (3 marks)

(c) Prove that $div(\mathrm{Curl}(\vec{A})) = 0$ (5 marks)

QUESTION THREE: (20 MARKS)

(a) Given the vector function $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y\sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$

(a) Show that $\vec{F}$ is a conservative force field (3 marks)

(ii) Find the scalar potential (4 marks)

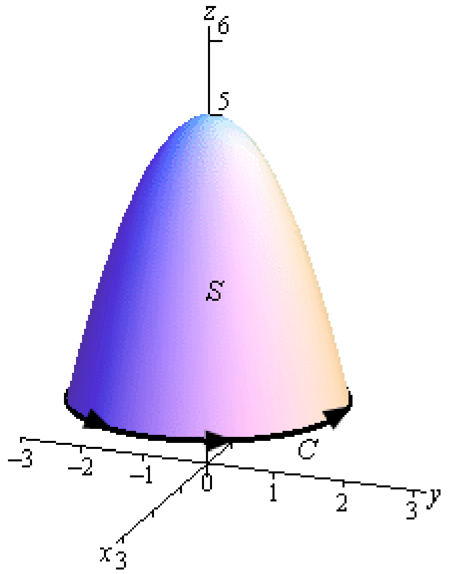
(iii) Evaluate $\int_C \vec{F}.\vec{dr}$, where c is a straight line from (0,1,-1) to $(\frac{\pi}{2}, -1, 2)$ (3 marks)

(b) State and Verify divergence theorem for $\vec{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 4, 0 \le z \le 3$ (10 marks)

QUESTION FOUR: (20 MARKS)

(a) Evaluate the line integral $\int_C y^3dx - x^3dy$ where C is the positively oriented circle of radius 2 centered at the origin. (10 marks)

(b) Use Stoke's theorem to evaluate $\iint_S \; curl\,\vec{F}\,\overrightarrow{ds}$, where $\vec{F} = z^2\vec{i} - 3xy\,\vec{j} + x^3y^3\vec{k}$ and S is the part of surface $z = 5 - x^2 - y^2$ above the plane $z = 1$. Assume that S is oriented upwards.



(10 marks)

QUESTION FIVE: (20 MARKS)

(a) Find the volume of the parallelepiped with adjacent sides

$\vec{u} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$, $\vec{v} = -\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$, $\vec{w} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ (4 marks)

(b) If $\vec{A} = x^2yz\hat{\imath} - 2xz^3\hat{\jmath} + xz^2\hat{k}$and $\vec{B} = 2z\hat{\imath} + y\hat{\jmath} - x^2\hat{k}$, find $\frac{\partial^2}{\partial_x \partial_y}\overrightarrow{(A\,X\vec{B})}$ at (1,0,-2)

(5 marks)

(c) Verify the Green's Theorem for $\int_C \; x^3dx + 2x(1 + xy)dy,$ where C is the unit circle.

(11 marks)
