CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF

MATH 221: CALCULUS II

STREAMS: **TIME: 2 HOURS**

DAY/DATE: TUESDAY 30/03/2021 **11.30 A.M – 1.30 P.M**

INSTRUCTIONS:

Answer question one and any other two questions

QUESTION ONE (COMPULSORY) (30 MARKS)

(a) Find the integral of the following functions

(i) $\int x^2 csc^2 x^3 Cot^4 x^3 dx$ [3 marks]

(ii) ${}^{d}/_{dx}\left(\int_x^{2x}(\frac{dt}{1+t^2}\right)$ [3 marks]

(iii) $\int \frac{dx}{x^2+2x+5}$ [3 marks]

(iv) $\int \frac{x}{\sqrt{x^2+2x-3}} dx$ [3 marks]

(b) Assuming that in a certain city the temperature (In°E) t hours after 9AM is represerited by the function

T(t) = 50 + 14 sin $\frac{\pi t}{12}$

Find the average temperature in the city during the period from 9AM to 9 P.M. [3 marks]

(c) By taking n = 4, find the approximate value of

$\int_0^2 \sqrt{4 + x^2}$ dx

Using the trapezoidal rule. [3 marks]

(d) Find the length of the curve

y = $\frac{4}{3}$ $\sqrt{2}\, x^{\frac{3}{2}}$ = 1 from x = 0 to x = 1 [3 marks]

(e) Find the Maclourin series of sine function. [3 marks]

(f) Evaluate $\int_1^4 \int_{-1}^2 (2x + 6x^2 y)\, dy\, dx$ [3 marks]

(g) Find the area of the region bounded by the graphs of y + $x^2 = 6$ and y + 2x – 3 = 0

[3 marks]

QUESTION TWO (20 MARKS)

(a) Approximate

$\int_1^2 \left(\frac{1}{x}\right) dx$ by using Simpson's rule with n = 10. Estimate the error in the approximation.

[5 marks]

(b) Evaluate $\int \frac{dx}{x(x^2+x=1)}$ [4 marks]

(c) Find the integral of $sin^n\, dx$ [5 marks]

(d) Evaluate the following definite integral of the f(x) such that

(i) f(x) = $\int_{-3/4}^{3/4} \frac{dx}{\sqrt{9-4x^2}}$ [3 marks]

(ii) $\int_0^2 \int_{x^2}^{2x} (x^3 + 4y) dy\, dx$ [3 marks]

QUESTION THREE (20 MARKS)

(a) Evaluate $\int \frac{(1-9x^2)^{\frac{3}{2}}}{x^4}\, dx$ [6 marks]

(b) Evaluate $\int tan^3 x dx$ [4 marks]

(c) Evaluate $\int \frac{3x^4+x^3+20x^2+3x+31}{(x+1)(x^2+4)^2}$ [4 marks]

(d) Prove that if f and g are continuous on [a,b] and g is non negative than there is a number c in (a,b) for which

$\int_a^b f(x)\, dx = f(x) \int_a^b g(x)\, dx$ [6 marks]

QUESTION FOUR (20 MARKS)

(a) Find $\int \sqrt{1 - \ e^x}$ dx [3 marks]

(b) Use trapezoidal rule to estimate the value of $\int_0^1 x \sin x\, dx$

Taking n = 10 . Find the error term associated with such approximation. [7 marks]

(c) If f(x) = $x^2 + 1$ find the volume of the solid generated by revolving the region under the graph of f from -1 to 1 about x – axis.

[3 marks]

(d) Integrate $\int x^2\, e^{2x}\ dx$

(e) Use reduction formula to evaluate

$\int sin^4 x\ \ dx$ [4 marks]

QUESTION FIVE (20 MARKS)

(a) Find the area bounded by the graphs of y –x = 6 and y -$x^3 = 0$ and 2y + x = 0.

[3 marks]

(b) Evaluate the following integral

(i)$\int \frac{3x^2+2x+3}{(x^2+1)^2}$ [4 marks]

(ii) $\int sin^4\ cos^2x\ \ dx$ [5 marks]

(iii) $\int e^x \cos 2x\ \ dx$ [5 marks]

(c) Find the distance travelled between t = 0 and t = $\frac{\pi}{2}$ by a particle whose position at time t is given by x = sin^2t, y = sin^2t [3 marks]
