CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT\SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE

MATH 401\403: MEASURE THEORY

STREAMS: BSC **TIME: 2 HOURS**

DAY/DATE: THURSDAY 12/8/2021 **2.30 P.M. – 4. 30 P.M.**

INSTRUCTIONS: ANSWER ALL QUESTIONS

QUESTION ONE (30 MARKS)

a) Show that a sigma algebra is closed under countable intersections (3 marks)

b) Prove that any interval of the form (a,∞) is Lebesgue measurable (4 marks)

c) Show that a constant function on a measurable set is measurable. (3 marks)

d) Define the characteristic function on a measurable subset E of R and show that it is measurable (4 marks)

e) Let $E_1 \subseteq E_2 \subseteq E_3 \subseteq$......... be a nested sequence of Lebesgue measurable sets and $E = \bigcup_{n=1}^{\infty} E_n$ show that $\lim_{n\to\infty} \mu(E_n) = \mu(E)$. (5 marks)

f) When do we say that a property holds measure almost everyhere? (1 mark)

g) Define a complete measure and show that the space (R, M, μ) of Borel measure space is complete. (4 marks)

h) Define the probability measure (2 marks)

i) let $f \in x$ and $f_1, f_2 \in M(x, X)$ such that $f = f_1 - f_2$ .suppose that $\int f_1 d\mu < \infty$ and $\int f_2 d\mu < \infty$, show that $\int f d\mu = \int f_1 d\mu - \int f_2 d\mu$ (4 marks)

QUESTION TWO (20 MARKS)

a) Define a Lebesgue measurable subset of R. (1 mark)

b) Show that if E is measurable, then its complement is also measurable. Hence or otherwise show that the sets Rand ϕ are measurable sets. (6 marks)

c) Show that if $\mu^*(A) = 0$, then A is measurable hence or otherwise show that a countable set is measurable. (6 marks)

d) Let A be a Lebesgue measurable subset of R, and B be any other subset of R. show that $\mu^*(A \cup B) + \mu^*(A \cap B) = \mu^*(A) + \mu^*(B)$. (7 marks)

QUESTION THREE (20 MARKS)

a) Let f and g be Lebesgue measurable function and c be a non-zero constant, show that $cf, c + f, f^2, |f|, f + g$, and fg are Lebesgue measurable. (12 marks)

b) Let f be a measurable function, prove that the following conditions are equivalent

   i. $\{x : f(x) > \alpha\}$ is Lebesgue measurable $\forall \alpha \in R$

   ii. $\{x : f(x) \geq \alpha\}$ is Lebesgue measurable $\forall \alpha \in R$

   iii. $\{x : f(x) < \alpha\}$ is Lebesgue measurable $\forall \alpha \in R$

   iv. $\{x : f(x) \leq \alpha\}$ is Lebesgue measurable $\forall \alpha \in R$ (8 marks)
