CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SICENCE IN

MATH 422: ODE II

STREAMS: **TIME: 2 HOURS**

DAY/DATE: MONDAY 20/09/2021 **8.30 A.M – 10.30 A.M**

INSTRUCTIONS

- **Answer question one (Compulsory) and any other two questions**

QUESTION ONE (30 MARKS)

(a) Show that $y_1 = e^t\ suit$ and $y_2 = e^t\ cost$ are a fundamental set. [4 marks]

(b) Solve the system. [5 marks]

(c) Determine whether the vectors function are linearly dependent (LD) or linearly independent (LI) . [3 marks]

(d) $\underline{x}_1 = \begin{pmatrix} e^{3t} \\ 0 \\ e^{3t} \end{pmatrix} \quad \underline{x}_2 = \begin{pmatrix} -e^{3t} \\ e^{3t} \\ 0 \end{pmatrix} \quad and \ \_\_\__3 = \begin{pmatrix} -e^{-3t} \\ -e^{-3t} \\ e - 3t \end{pmatrix}$

Are a fundamental set for a system $\underline{x}^1 = Ax + fct$ whose particular integral is $\begin{pmatrix} 5t+1 \\ 2t \\ 4t+2 \end{pmatrix}$

Write down specific /particular solution to the system. [3 marks]

(e) Convert the differential equation with a system of first order differential equations in matrix form. [5 marks]

$$\frac{d^4y}{dx^4} - \frac{7d^3y}{dx^3} + \frac{4d^2y}{dx^2} + \frac{5dy}{dx} - 2y = 0$$

(f) Show that the singular point of the equation below is regular.

$(x-3)^2y'' + 2(x-3)y' + 4y = 0$ [4 marks]

QUESTION TWO (20 MARKS)

(a) The general Bessel's equation is $x^2y'' + xy' + (x^2 - t^2)y = 0$

(i) Write down the Bessel's equation of order ½. [2 marks]

(ii) Determine and classify the singular points of the Bessel's equations. [4 marks]

(iii) State the significance of classifying the singular points of a differential equation. [3 marks]

(b) Solve the differential equation $y'' + xy'y = 0$ for a series solution at $x_0 = 0$. [8 marks]

(c) Solve the initial value problems $\begin{cases} x^1 = A\,\underline{x} \\ x_{1,}(0) = 1x_2(0) = 2, x_3(0) = 3 \end{cases}$ given that

$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 0 & 4 & 1 \end{pmatrix}$ [8 marks]

QUESTION THREE (20MARKS)

(a) (i) State the Rodrigues formula for finding the Legendre polynomials $p_n(x)$. [1 mark]

(ii) Use the Rodrigues formula stated in a(i) above to obtain $p_5(x)$. [6 marks]

(b) Find the power series of the D.E $y' - 2y = 0$ [8 marks]

(c) Reduce the equation

$x''' + 2x'' + 5x' + 7x = 0$ into a system of linear first order equations in matrix form. [5 marks]

QUESTION FOUR (20 MARKS)

(a) The Legendre polynominals $p_n(x)$ are given by

$$P_n(x) = \frac{1}{2^n}\sum_{k=0}^{n/2} \frac{(-1)^k(2n-2k)^1\ x^{(n-2k)}}{k^!(n-k)^!(n-2k)^!}$$

$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$ [5 marks]

(b) Solve the initial value problem.

$x^1(t) = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix} \underline{x(t)}$ given that $x_0 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ [8 marks]

(c) Use systematic elimination to solve

$\frac{dx}{dt} = 2x - y$ [7 marks]

$\frac{dy}{dt} = x$
