CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF SCIENCE IN MATHEMATICS AND BACHELOR OF EDUCATION (SCIENCE)

MATH 427: PARTIAL DIFFERENTIAL EQUATIONS II

STREAMS: BSC. MATHS & B. ED SCI. **TIME: 2 HOURS**

DAY/DATE: MONDAY 27/09/2021 **8.30 A.M. – 10.30 A.M.**

INSTRUCTIONS:

- **Answer question one and any other two questions**
- **Adhere to the instructions on the answer booklet.**

QUESTION ONE Compulsory

a. Given the Partial differential equation $(U_{xz})^3 + U_{xyz} = U_x$ State the order, linearity and degree of the Pde giving reasons (3 marks)

b. Evaluate $U_{xy} = e^y \cos x$ given that $U(0, y) = -e^y \sin x$ by direct integration (5marks)

c. Given the PDE, $(3+y)U_{xx} + 2(3-x)U_{xy} + (3+y)U_{yy} = U_x + U_y$, determine the values of (x) and (y) for which the equation is

   i. Hyperbolic. (2 marks)

   ii. Parabolic (2 marks)

d. Solve the pde $U_{xx} - 4U_{xy} + U_{yy} = 0$ by the D operator (3 marks)

e. Apply the method of separation of variables to solve $U_x = 2U_t + U$, given that

   $U(x,0) = 6e^{-3x}$ (5 marks)

f. Given the partial differential equation $U_{xx} + 4U_{xy} + 5U_{yy} = 0$

i. Classify the pde (2 marks)

ii. Find the characteristics (4 marks)

iii. Obtain U_{xx} and U_{yx} in terms of the characteristics (4 marks)

QUESTION TWO

a. Classify the following PDE's

(i). $U_{tt} = 4U_{xx}$ (2 marks)

(ii). $2U_{t} = 3U_{xx}$ (2 marks)

(iii). $U_{xx} - 4U_{xy} + U_{yy} = 0$ (2 marks)

b. Solve the equation $U_{t} = U_{xx}$ with boundary conditions given that $0 < x < l$

7marks

$U(0,t) = 0$
$U(l,t) = 0$
$U(x,0) = 3\sin n\pi x$

c. Solve the pde $U_{xx} - U_{xy} = \sin x \cos 2y$ by the D operator (7 marks)

QUESTION THREE

a. Given that the homogeneous PDE $A(x,y)U_{xx} + 2B(x,y)U_{xy} + C(x,y)U_{yy} = 0$, find it's characteristic curve λ, and show that $\lambda = \frac{-B}{C}$ if it is parabolic (4 marks)

b. Classify the following differential equation in the second quadrant of the xy plane.

$$\left(\sqrt{y^2 + x^2}\right)U_{xx} + 4(x-y)U_{xy} + \left(\sqrt{y^2 + x^2}\right)U_{yy} = 0$$ (4 marks)

c. Solve the pde $U_{xx} - 2U_{xy} + U_{yy} = \sin x$ by the D operator (6 marks)

d. Evaluate $U_{xy} = x^2 y$ given that $U(x,0) = x^2$ and $U(1,y) = \cos y$ by direct integration (6 marks)

QUESTION FOUR

a. Solve the pde $U_{xx} + U_{xy} - 6U_{yy} = y\cos x$ by the D operator (6 marks)

b. Solve the non-homogeneous pde $U_{xx} + U_x - U_{yy} + 3U_y - 2U = x^2 y$ (7 marks)

c. Solve the pde $U_{xx} + U_{yy} = x^2 y^2$ by the D operator (7 marks)

QUESTION FIVE

a. Apply the method of separation of variables to solve $U_x = 2U_t + U$, given that

$U(x,0) = 0$ and $U_t(0,t) = 0$ (10 marks)

b. The vibrations of an elastic string is governed by the equation $U_{xx} = U_{tt}$. Find the deflection U(x,t) of the vibrating string for t > 0, under the following conditions.

(10 marks)

$U(0,t) = 0$

$U(\pi,t) = 0$

$U_t(x,0) = 0$

$U(x,0) = 2(\sin x + \sin 3x)$

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