## CHUKA

UNIVERSITY EXAMINATIONS

## RESIT/SPECIAL EXAMINATION

## EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF

## MATH 442: TIME SERIES ANALYSIS

STREAMS:
TIME: 2 HOURS

DAY/DATE: TUESDAY 10/08/2021
11.30 A.M - 1.30 P.M.

## INSTRUCTIONS

- ANSWER ALL THE OUESTIONS.


## Question One (30 marks)

a) Find the Yule Walker equations of the AR (3) process $X_{t}=\emptyset_{1} X_{t-1}+\emptyset_{2} X_{t-2}+$ $\emptyset_{3} X_{t-3}+e_{t}$ (5 marks)
b) Consider the AR (2) process given by $X_{t}=\frac{4}{5} X_{t-1}-\frac{15}{100} X_{t-2}+e_{t}$. Show that $X_{t}$ is stationary and find its autocorrelation function.
c) Determine whether the process $X_{t}=0.6 \mathrm{X}_{\mathrm{t}-1}-0.5 \mathrm{X}_{\mathrm{t}-2}+\mathrm{e}_{\mathrm{t}}-0.4 \mathrm{e}_{\mathrm{t}-1}+0.2 \mathrm{e}_{\mathrm{t}-2}$ is stationary and invertible
(5 marks)
d) Fit local polynomial of degree 3 to seven consecutive data point of weight $\mathrm{W}=1 / 21[-2,3,6,7,6,3,-2]$
e) Given the following observation of a time series $X_{t}$ for $n=10$.

| t | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{x}_{\mathrm{t}}$ | 47 | 64 | 23 | 71 | 38 | 64 | 55 | 41 | 59 | 48 |

Find
i. Sample auto-covariance $[\mathrm{r}(1)]$ and sample autocorrelation [rho(1)] (5 marks)
ii. Sample auto-covariance $[\mathrm{r}(2)]$ and sample autocorrelation $[\mathrm{rho}(2)]$ (5 marks)

## Question Two (20 marks)

The data below give the average quarterly prices of a commodity for four(4) years

| YEAR | I | II | III | IV |
| :--- | :--- | :--- | :--- | :--- |
| 2011 | 50.4 | 40.8 | 47.0 | 49.2 |
| 2012 | 38.4 | 33.6 | 53.2 | 69.5 |
| 2013 | 67.2 | 53.2 | 60.7 | 42.6 |
| 2014 | 55.1 | 56.0 | 61.6 | 65.3 |

Calculate the seasonal indices using multiplicative model
(10 marks)
b) Find the spectral density function of the process

$$
\begin{equation*}
\mathrm{X}_{\mathrm{t}}+\mathrm{X}_{\mathrm{t}-1}+\mathrm{X}_{\mathrm{t}-2}=\mathrm{e}_{\mathrm{t}} \tag{5marks}
\end{equation*}
$$

c)Find the Yule Walker equations of the $\operatorname{AR}(3)$ process $X_{t}=\emptyset_{1} X_{t-1}+\emptyset_{2} X_{t-2}+\emptyset_{3} X_{t-3}+e_{t}$
(5 marks)

## Question Three (20 marks)

a) Consider the AR (2) process given by $X_{t}=\frac{4}{5} X_{t-1}-\frac{15}{100} X_{t-2}+e_{t}$. Show that $X_{t}$ is stationary and find its autocorrelation function.
(10 marks)
b) Determine whether the process $\mathrm{X}_{\mathrm{t}}=0.6 \mathrm{X}_{\mathrm{t}-1}-0.5 \mathrm{X}_{\mathrm{t}-2}+\mathrm{e}_{\mathrm{t}}-0.4 \mathrm{e}_{\mathrm{t}-1}+0.2 \mathrm{e}_{\mathrm{t}-2}$ is stationary and invertible.
(10 marks)

