

CHUKA



UNIVERSITY

## UNIVERSITY EXAMINATIONS

**FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR  
OF SCIENCE IN ARTS AND EDUCATION**

**MATH 442: TIME SERIES ANALYSIS**

**STREAMS: BSC, B.ED, BA  
HOURS**

**TIME: 2****DAY/DATE: THURSDAY 23/09/2021****8.30 A.M – 10.30 A.M.****INSTRUCTIONS:**

- Answer question ONE and any other TWO questions.

**QUESTION ONE (30 MARKS)**

- (a) Outline four (4) uses of time series analysis in statistics. (4 marks)
- (b) Consider the AR process and show that  $X_t = 0.8X_{t-1} - 0.15X_{t-2} + e_t$  (8 marks)
- (c) Given the following observation of a time series for  $n = 10$

$t$	1	2	3	4	5	6	7	8	9	10
$X_t$	0.812	1.657	2.537	3.431	4.329	5.254	6.174	7.104	8.044	8.956

Find

- (i) Sample auto-covariance  $r(1)$  and  $r(2)$  (4 marks)
- (ii) Sample auto-correlation  $p(1)$  and  $p(2)$  (4 marks)

Consider MA (2) process given by

$$X_t = e_t - 0.1e_{t-1} + 0.21e_{t-2}$$

- (i) Show that the model is stationery (4 marks)
- (ii) Show that the model is invertible (2 marks)
- (iii) Find the ACF for this process (4 marks)

**QUESTION TWO (20 MARKS)**

- (i) Find the Yule Walker equation of the AR (3) process

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \phi_3 X_{t-3} + e_t \quad (10 \text{ marks})$$

- (ii) Consider a set of independent and identically distributed random variable  $\{e_t\}$  such that

$E(e_t) = 0$  and variance of  $e_t$  is  $\sigma_e^2$ . Let the process be given by  $X_t = \phi e_{t-1} + e_t$  where  $\phi$  is a constant. Show that  $X_t$  is weakly stationary. (10 marks)

**QUESTION THREE (20 MARKS)**

- (i) Outline four (4) components of a time series data. (8 marks)
- (ii) Fit a local polynomial of degree 2 with 5 consecutive data points given the weight.

$$W = \frac{1}{35}(-3, 12, 17, 12, -3) \quad (12 \text{ marks})$$

**QUESTION FOUR (20 MARKS)**

- (i) Determine whether the process

$$X_t = 0.6X_{t-1} - 0.5X_{t-2} + e_t - 0.4e_{t-1} + 0.2e_{t-2}$$

Is stationary and invertible

- (ii) Find the covariance generating function of the MA (2) process given by

$$X_t = e_t + \frac{3}{10}e_{t-1} - \frac{1}{4}e_{t-2} \quad (6 \text{ marks})$$

Hence the autocorrelation function. (6 marks)

**QUESTION FIVE**

Find the spectral density function of the process (20 marks)

$$X_t + X_{t-1} + X_{t-2} = e_t$$