CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS
RESIT EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION SCIENCE AND BACHELOR OF SCIENCE

PHYS 232: WAVES AND OSCILLATIONS

STREAMS: BED (SCI) AND BSC **TIME: 2 HOURS**

DAY/DATE: TUESDAY 04/11/2018 **2.30 P.M – 4.30 P.M**

INSTRUCTIONS:

- Answer Question One and any other Two Questions.
- Do not write anything on the question paper
- This is a closed book exam, No reference materials are allowed in the examination room
- There will be No use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely
- You may use the data below.
    i. Gravitational Constant: $g = 10\ ms^{-2}$
    ii. $\sin(A+B)+\sin(A-B)=2\sin A\cos B$

QUESTION ONE (30 Marks)

a. Define the following terms;
   i. Simple harmonic motion **(3 marks)**
   ii. Displacement and Amplitude **(2 marks)**
   iii. normal modes of oscillation **(1 mark)**
   iv. Logarithmic decrement **(1 mark)**
   v. Resonance **(1 mark)**
   vi. Degrees of freedom **(1 mark)**

b. Write down the linear differential equation of motion for a free simple harmonic oscillator:
   i. In terms of m, the oscillator mass and k, the oscillator stiffness, **(1 mark)**
   ii. In terms of ω_n. **(1 mark)**

c. List the three mechanisms responsible for energy loss of a harmonic oscillator. Which mechanism is used in our analysis of free damped simple harmonic motion and why? **(4 marks)**

d. You are given the linear second order differential equation with constant coefficients m, k,

$$m\ddot{x} + c\dot{x} + kx = 0$$

and c, .

$$x = \exp(rt)$$

i. If the trial solution is then write down the characteristic equation of the above differential equation. **(1 mark)**

ii. Solve the above characteristic equation in r that you have written, and obtain an expression for the roots of this characteristic equation in terms of m, k, and c. **(1 mark)**

$$\xi = \frac{c}{2m\omega_n} \text{ and } \omega_n^2 = \frac{k}{m}$$

iii. Let us define two quantities then rewrite the expression for the roots,

ω_n and ξ

r, in terms of . **(2 marks)**

e. The displacement of a free harmonic oscillator can be expressed as x(t) = Asin(ωt+φ); Use the displacement function to show that the acceleration amplitude = $A\omega^2$ **(2 marks)**

f. What is a wave and how can we classify waves generally according how they are propagated by media? **(3 marks)**

g. A steel wire of 0.01 kg mass and 2 m length is stretched to a tension of 10 N.

i. Calculate the linear frequency of the fundamental vibrations. **(3 marks)**

ii. If the displacement amplitude of the fundamental is 0.02 m at the center of the wire, what is the total energy of the fundamental mode of vibration? **(1 mark)**

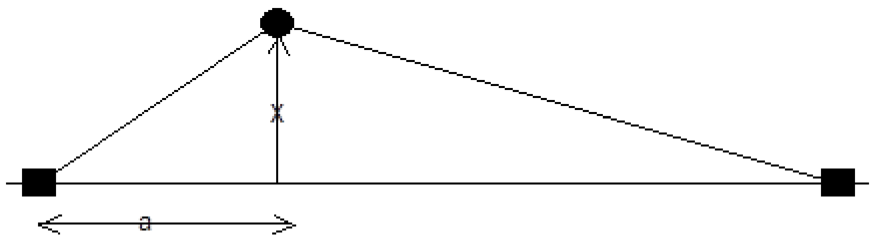
iii. What is the velocity amplitude at a point 0.5 m from either end of the wire? **(2 marks)**

<u>QUESTION TWO (20 Marks)</u>

a. What can you generally say about the energy in a free simple harmonic oscillator? **(1 mark)**

b. Distinguish between the phase constant, the phase angle, and the phase difference. **(3 marks)**

c. A mass m is fastened to a string of length l as shown below. The string is stretched to a tension T between two rigid supports.

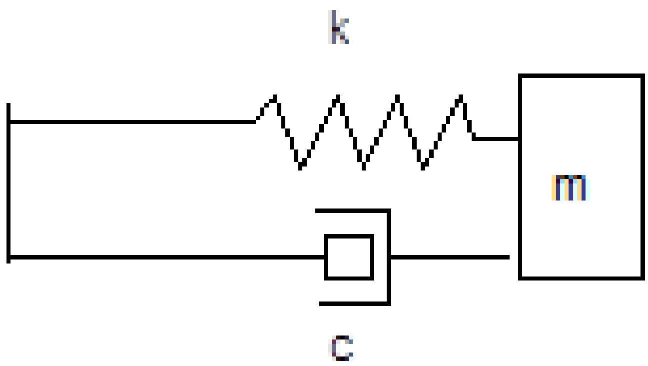


i. When the mass is displaced a small distance x as shown, what is the restoring force? (assume T is constant and neglect the effect of gravitational forces.) **(10 marks)**

ii. Derive an expression giving the linear frequency of the vertical vibration of the mass for such small amplitudes that the sines and the tangents of the angles between the segments of the string and the horizontal may be considered equal. **(6 marks)**

QUESTION THREE (20 Marks)

a. An industrial buffer unit comprises a piston of mass 5 kg, a spring of stiffness 2 kNm^{-1} and a damper that provides a linear damping force of 240 N at 1 ms^{-1}. The unit forms a mass-spring-damper system with a single degree of freedom as shown below.



The spring is initially compressed by 20 mm.

i. Obtain the differential equation for the motion of the piston as it moves to the left in free motion and hence determine the damping ratio of the system. **(3 marks)**

ii. If a sudden impulse of 10 Ns is applied to the piston, determine the distance moved by the piston before it momentarily comes to rest. **(12 marks)**

b. An instrument consists of mass of 80 g whose movement is controlled by a spring and a viscous damper. A free damped vibration of periodic time 0.5 s gives the following readings for successive displacements on either side of the equilibrium position at which the reading is 60.

Readings	70	55	62.5	58.75	60.62

For these readings determine:

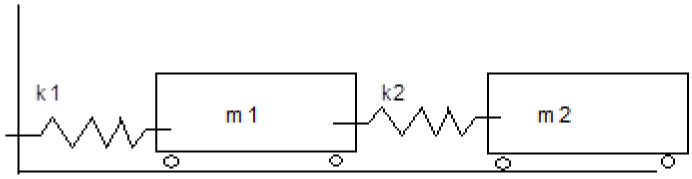
i. The damping ratio **(3 marks)**

ii. The stiffness of the spring **(3 marks)**

iii. The force exerted by the damper at a speed of 1 ms^{-1}. **(1 mark)**

QUESTION FOUR

Consider the following system of coupled free oscillators below.



a. Consider the body m_1. By applying Newton's second law obtain the equation of motion for this body. **(4 marks)**

b. Now apply Newton's second law to obtain the equation of motion for m_2 . **(3 marks)**

c. Given that $x_1 = A_1 \sin \omega t$ and $x_2 = A_2 \sin \omega t$ derive the frequency equation and hence obtain an expression for ω^2 . **(6 marks)**

d. Now if m_1= 2 kg, m_2 = 3 kg, k_1= 2 kNm^{-1}, and k_2 = 4 kNm^{-1}, then calculate the natural angular frequencies of this system **(3 marks)**

e. Determine their respective amplitude ratios. **(4 marks)**

QUESTION FIVE (20 Marks)

Consider an element dx of string of length L, which is deformed to length ds by the application of a tension T. if the string has a mass per unit length of ρ .

a. Derive expressions for the kinetic energy per wavelength K_λ and the potential energy per wavelength U_λ , and hence write down the expression for the total energy per wavelength E_λ . **(8 marks)**

hint: you may use the identity $ds = \left[(dx)^2 + (dy)^2\right]^{\frac{1}{2}} \approx dx\left[1 + \left(\frac{\partial y}{\partial x}\right)^2\right]^{\frac{1}{2}} \approx 1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2$

b. If a wave is described by $y(x,t) = A\sin 2\pi\nu\left(t - \frac{x}{c}\right)$ for a string, show that the mean power input $P = \frac{u_0^2 T}{2c}$. Given that $\int_{t=0}^{t=\tau} \cos^2 2\pi\nu t\, dt = \frac{1}{2}\tau$, $\int_{t=0}^{t=\tau} dt = \tau$ and

$$P = \frac{\int_{t=0}^{t=\tau} P_{inst}\, dt}{\int_{t=0}^{t=\tau} dt}$$

(12 marks)
