CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF

MATH 204: ALGEBRAIC STRUCTURES

STREAMS: **TIME: 2 HOURS**

DAY/DATE: WEDNESDAY 11/08/2021 **2.30 P.M – 4.30 P.M.**

INSTRUCTIONS
Answer ALL the questions.

QUESTION ONE (30 MARKS)

a) Define the following terms
   i. A subgroup H of a group G (1 mark)
   ii. A homomorphism of groups (1 mark)
   iii. The characteristic of a ring (1 mark)
   iv. A principal ideal (1marks)

b) Let S be a set of four elements given by $S = \{A, B, C, D\}$. In the table below, all the elements of S are listed in a row at the top and in a column at the left. The result $x * y$ is found in the row that starts with x at the left and the column that has y at the top.

*	A	B	C	D
A	B	C	A	B
B	C	D	B	A
C	A	B	C	D
D	A	B	D	D

   i. Is the binary operation * commutative? Support your answer. (2 marks)
   ii. Determine whether there is an identity element in S for * (2 marks)
   iii. If there is an identity element, which elements in S are invertible? (2 marks)

c) Given a group G, define the centre of G,(z(G)) and show that it is a normal subgroup of G (5 marks)

d) Let G be a cyclic group generated by a i.e. $G = < a >$.Prove that G is abelian. (5 marks)

e) The addition and part of the multiplication table for the ring R={a,b,c,d} are given below. Use the distributive laws to complete the multiplication table below (5 marks)

+	A	B	C	D
A	A	A	C	D
B	B	C	D	A
C	C	D	A	B
D	D	A	B	C

.	A	B	C	D
A	A	A	A	A
B	A	C		D
C	A		A	
D	A		A	C

f) Let I be an ideal of a ring R, prove that the set $K = \{x \in R : xa = 0 \forall a \in R\}$ is an ideal of R (5 marks)

QUESTION TWO (20 MARKS)

a) Consider the set $R = \{[0],[2],[4],[6],[8]\} \subseteq Z_{10}$.

i. Construct addition and multiplication tables for R using operations as defined in Z_{10} (4 marks)

ii. Show that R is a commutative ring with unity. (2 mars)

iii. Show that R a subring of Z_{10} (2 marks)

iv. Does R have zero divisors? (2 marks)

v. Is R an integral domain? (2 marks)

vi. Is R a field? (2 mark)

b) Consider the group $D_3 =< a,b : a^2 = b^3 = e; ba = ab^2 >$ and the subgroup $H = \{e,a\}$.

i. List the right and left cosets of H in D_3 (5 marks)

ii. Is H a normal subgroup of D_3? Support your answer. (1 marks)

QUESTION THREE (20 MARKS)

a) State and prove Lagrange's theorem. (7 marks)

b) Given that the set $S = \left\{ \begin{bmatrix} x & y \\ 0 & z \end{bmatrix} \middle| x, y, z \in Z \right\}$ is a ring with respect to matrix addition and multiplication, show that $I = \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \middle| a,b \in Z \right\}$ is an ideal of S (7 marks)

c) Prove that he characteristic of an integral domain is either zero or a prime integer (6 marks)
