CHUKA UNIVERSITY

SUPPLEMENTARY / SPECIAL EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF BACHELOR DEGREE

MATH 204 : ALGEBRAIC STRUCTURES

STREAMS:

TIME: 2 HOURS

DAY/DATE: MONDAY 16/11/2020 **2.30 P.M - 4.30 P.M.**

INSTRUCTIONS:
Answer All Questions

QUESTION ONE (30 MARKS)

a) Define the following terms
   i. A subgroup H of a group G (1 mark)
   ii. A homomorphism of groups (1 mark)
   iii. The characteristic of a ring (1 mark)
   iv. A principal ideal (1marks)

b) Let S be a set of four elements given by $S = \{A, B, C, D\}$. In the table below, all the elements of S are listed in a row at the top and in a column at the left. The result $x * y$ is found in the row that starts with x at the left and the column that has y at the top.

*	A	B	C	D
A	B	C	A	B
B	C	D	B	A
C	A	B	C	D
D	A	B	D	D

i. Is the binary operation * commutative? Support your answer. (2 marks)
ii. Determine whether there is an identity element in S for * (2 marks)

iii. If there is an identity element, which elements in S are invertible? (2 marks)

c) Given a group G, define the centre of G,(z(G)) and show that it is a normal subgroup of G (5 marks)
d) Let G be a cyclic group generated by a i.e. $G=<a>$.Prove that G is abelian. (5 marks)
e) The addition and part of the multiplication table for the ring R={a,b,c,d} are given below. Use the distributive laws to complete the multiplication table below (5 marks)

+	A	B	C	D
A	A	A	C	D
B	B	C	D	A
C	C	D	A	B
D	D	A	B	C

.	A	B	C	D
A	A	A	A	A
B	A	C		D
C	A		A	
D	A		A	C

f) Let I be an ideal of a ring R, prove that the set $K=\{x \in R: xa=0 \forall a \in R\}$ is an ideal of R (5 marks)

<u>QUESTION TWO (20 MARKS)</u>

a) Consider the set $R=\{[0],[2],[4],[6],[8]\} \subseteq Z_{10}$.
   i. Construct addition and multiplication tables for R using operations as defined in Z_{10} (4 marks)
   ii. Show that R is a commutative ring with unity. (2 mars)
   iii. Show that R a subring of Z_{10} (2 marks)
   iv. Does R have zero divisors? (2 marks)
   v. Is R an integral domain? (2 marks)
   vi. Is R a field? (2 mark)
b) Consider the group $D_3 =< a,b : a^2 = b^3 = e; ba = ab^2 >$ and the subgroup $H=\{e,a\}$.
   i. List the right and left cosets of H in D_3 (5 marks)
   ii. Is H a normal subgroup of D_3? Support your answer. (1 marks)

QUESTION THREE (20 MARKS)

a) Given the set $A = \{5,15,25,35\}$

   i. Show that A is a group under multiplication modulo 40 (8 marks)

   ii. Find the identity element (2 marks)

b) Identify which of the following maps are group homomorphism and if it is, find its kernel

   i. G is the group of non-zero real numbers under multiplication and $\varphi(x) = 2^x \; \forall x \in G$

   ii. G is the group of real numbers under addition and $\phi(x) = x + 1$ (6 marks)

b) Prove that a group G is abelianiff $(ab)^{-1} = a^{-1}b^{-1} \; \forall a, b \in G$ (4 marks)

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