CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF

BACHELOR OF SCIENCE DEGREE IN MATHEMATICS AND BACHELORS OF SCIENCE CHEMISTRY

MATH 204: ALGEBAIC STRUCTURES

STREAMS: AS ABOVE

TIME: 2 HOURS

DAY/DATE: WEDNESDAY 31/3/2021 **11.30 AM – 1.30 PM**

INSTRUCTIONS:
- Answer Question **ONE** and any other **TWO** Questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE (30 MARKS)

a) Given that $x, y \in N$ Determine whether the following binary operations are commutative, associative.

i. $x * y = x + xy$ (3 marks)

ii. $x * y = \frac{xy}{x+y}$ where $x \neq -y$ (3 marks)

b) Let G be the group of integers under addition and G' be the group with elements {1,-1} under multiplication. Define a mapping $\emptyset(x) = \begin{cases} 1 \; if \; n \; is \; odd \\ -1 if \; n \; is \; even \end{cases}$

i. Show that ∅ is a group homomorphism (3 marks)

ii. Find ker∅ (2 marks)

c) Verify whether or not the following statements are true about groups
   i. A group of order 32 has a subgroup of order 18 (3 marks)
   ii. Every cyclic group is abelian (3 marks)

d) Let G be a group and $a,b,c \in G$, show that $a \circ b = a \circ c$ imply $b = c$ (3 marks)

e) The addition and part of the multiplication table for the ring R={a,b,c,d} are given below. Use the distributive laws to complete the multiplication table below .

+	A	B	C	D
A	A	A	C	D
B	B	C	D	A
C	C	D	A	B
D	D	A	B	C

.	A	B	C	D
A	A	A	A	A
B	A	C		D
C	A		A	
D	A		A	C

(4marks)

e) If θ is a homomorphism of rings, show that
   i. $\theta(0) = 0$ (3 marks)
   ii. $\theta(-r) = -\theta(r)$ for all r (3 marks)

QUESTION TWO (20 MARKS)

a) (i)Write the permutations (23) and (13)(245) on 5 symbols in two line notation.
   (ii) Express the products (23)$\circ$ (13)(245) and (13)(245) $\circ$(23) in cyclic notation.
   (iii) Express in cyclic notation the inverses of (23) and of (13)(245). (6 marks)

b) Show that the set $S = \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}, \begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}, \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\right\}$forms a cyclic group under matrix multiplication. Construct a Carley table for the group (6 marks)

c) Consider the the set S={a,b,c,d}.Verify whether or not S together with each of the binaries represented in the tables below forms a commutative group on the set S. (assume associativity)

•	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

*	a	b	c	d
a	d	a	c	b
b	a	c	b	d
c	b	d	a	c
d	c	b	d	a

(8 marks)

QUESTION THREE (20 MARKS)

a) Given a group G, define the centre of G,(z(G)) and show that it is a normal subgroup of G (4 marks)

b) Let G be a group of order 91. Verify whether or not G is cyclic.
(Hint: 91 is prime) (4 marks)

c) Given the set $A = \{5,15,25,35\}$Show that A is a group under multiplication modulo 40 (5 marks)

d) Consider the group $D_4 = < a,b : a^2 = b^4 = e; ba = ab^3 >$ and the subgroup $H = \{e, b^2\}$.

i. List the right and left cosets of H in D_4 (5 marks)

ii. Is H a normal subgroup of D_4? Explain (2 marks)

QUESTION FOUR (20 MARKS)

a) Show that $S = \{2k : k \in Z\}$ with addition and multiplication as defined on Z is a ring while $T = \{2k+1 : k \in Z\}$ is Not (4 marks)

b) Define and give an example of an integral domain. Suppose a, b and c are elements of an integral domain D such that ab=ac and $a \neq 0$. Prove that b=c (5 marks)

c) Consider the set $R = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \bar{.0}.1\} \subseteq Z_{14}$.(some of the remainders of division modulo 14)

i. Construct addition and multiplication tables for R using operations as defined in Z_{14} (4 marks)

ii. Show that R is a commutative ring with unity. (2 mars)

iii. Show that R a subring of Z_{14} (2 marks)

iv. Does R have zero divisors? (1 marks)

v. Is R a field? If yes illustrate each element with its inverse (2 mark)

QUESTION FIVE (20 MARKS)

a) Let G be a group in which every element has order at most 2. Show that G is abelian (3 marks)

b) Show that in an abelian group G, the set of all elements with finite order in G is a subgroup of G. (5 marks)

c) Let G be the set of eight elements given by $G = \{\pm 1, \pm i, \pm j, \pm k\}$ with multiplication given by $i^2 = j^2 = k^2 = -1$, $ij = -ji = k$, $jk = -kj = i$, $ki = -ik = j$.

i. Construct a multiplication table for the group. (6 marks)

ii. Consider the cyclic group group $H = < i >$. list all the distinct cosets of H in G. Is H a normal subgroup of G? (6 marks)

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