CHUKA **UNIVERSITY**

UNIVERSITY EXAMINATION

RESIT /SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE

MATH 205: ELEMENTS OF SET THEORY

STREAMS: **TIME: 2 HOURS**

DAY/DATE: THURSDAY 04/11/2021 **8.30 A.M – 10.30 A.M**

INSTRUCTIONS:

Answer all the questions

QUESTION ONE (30 MARKS)

a) For each of the following cases, determine whether it represents a function. If it is a function, state whether it is injective
   i. To each of the 24 student in math 205, assign the gender
   ii. To each student in Chuka university, assign a registration number
   iii. To each student in first year, assign the semester course units
   iv. To each book written by a single author, assign the author
   v. To each positive number, assign its square root (5 marks)

b) Given the sets $A_n = \{n, n+2\}$, where n is a positive integer evaluate
   i. $\bigcup_{n=3}^{10} A_n$ and $\bigcap_{n=1}^{10} A_n$
   ii. $\bigcup_n A_n$ and $\bigcap_n A_n$ (4 marks)

c) Find the domain of the function $f: R \to R$ defined by $f(x) = \dfrac{4}{\sqrt{x^2 - 4}}$ (5 marks)

d) Consider the set $A = \left\{ 11 + (-1)^n \dfrac{1}{n} \right\}$ where n is a positive integer
   i. Find the supremum and the infimum of A (2 marks)
   ii. Find all the limit points of A (1 marks)

e) With an appropriate example, show that a bounded sequence is not necessarily convergent (3 marks)

f) Let A and B be sets. Show that the product order on $A \times B$ defined by $(a,b) \prec (c,d)$ if $a \le c$ and $b \le d$ is a partial order on $A \times B$ (4 marks)

g) Prove that the set of integers is countable (4marks)

h) State the Axiom of choice (2 marks)

QUESTION TWO (20 MARKS)

a) Distinguish the following

i. A restriction and an inclusion map (3 marks)

ii. A countable and uncountable set (3 marks)

iii. A linearly ordered set and a poset (3 marks)

b) Prove the generalized Consider the function $f : R \to R$ defined by $f(x) = \frac{|x|}{x} : x \neq 0$ and $f(0) = 0$. Determine

i. The quotient sets $\frac{R}{f}$ (3 marks)

ii. The image $f(R)$ (3 marks)

c) Prove that if the limit of a sequence exists, then it is unique (5marks)

QUESTION THREE (20 MARKS)

a) Let $A_m = \{m, 2m, 3m, ...: m \in N\}$, determine and explain the following sets

i. $A_3 \cap A_7$ (3 marks)

ii. $A_3 \cup A_7$ (3 marks)

iii. $\bigcup_m A_m$ (2 marks)

iv. $\bigcap_m A_m$ (2 marks)

b) Prove that the intervals [0,1]and(0,1] are equivalent. (5 marks)

c) Prove that a countable union of finite sets is countable (5 marks)
