CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

SECOND YEAR EXAMINATION FOR THEAWARD OF DEGREE OF BACHELORS OF SCIENCE IN MATHEMATICS & BA MATHS-ECONS

MATH 206: INTRODUCTION TO REAL ANALYSIS

STREAMS: ````as above```` Y2S2 **TIME: 2 HOURS**

DAY/DATE: TUESDAY 10/08/2021 **2.30 P.M. – 4.30 P.M.**

INSTRUCTIONS:

- Answer question **ALL** the questions.
- Sketch maps and diagrams may be used whenever they help to illustrate your answer.
- Do not write on the question paper.
- Write your answers legibly and use your time wisely.

QUESTION ONE: 30 MARKS

(a) Find if the following sets are bounded or not and if bounded find the $sups$ and $infs$

(i) $S_1 = \{x \in \mathbb{R}: 3 \le x < 7\}$ (4 marks)

(ii)$S_2 = \left\{1 + (-1)^n \frac{1}{n}: n \in \mathbf{N}: \right\}$ (4 marks)

(iii) $S_2 = \{1 + (-1)^n . n: n \in \mathbf{N}: \}$ (4 marks)

(b) Determine the accumulation points of each of the set of real numbers

(i) The set of natural numbers **N**;

(ii) $(a, b]$

(iii) The set of irrational points (4 marks)

(c) Let $A \subseteq \mathbb{R}$ be given by $A = \{x \in \mathbb{R}: 0 < x \le 1\}$. Show that the element $\frac{1}{2} \in A$ is an interior point of A whereas 1 is not (4 marks)

(d) Prove that if a function is differentiable at a point $x = a$ then the function is also continuous at the same point. (4 marks)

(e) Show that the subset $A \subset \mathbb{R}$ is closed if and only if $A = \bar{A}$ (4 marks)

(f) State without proof the following properties for real numbers.
(i) Completeness axiom (2 marks)
(ii) Archimedean Property (1 marks)

QUESTION TWO: (20 MARKS)

(a) Given that $A \subseteq \mathbb{R}$, define an interior point x of A. Hence show that if A is open if and only if A is equal to its interior set A^0 (5 marks)

(b) Prove that a limit of function exists then that limit is unique (4 marks)

(c) Denote the closure of a subset $B \subset \mathbb{R}$ by $\bar{B}$. Prove that $B = \bar{B}$ if and only if B is closed (5 marks)

(d) Find the limit superior and limit inferior of the sequence

$X_n = \left(1 + \frac{n}{n+1} + cos\ \frac{n\pi}{2}\right) : n \in \mathbf{N}$ (6 marks)

QUESTION THREE: (20 MARKS)

(a) Define a Cauchy sequence (x_n) in R. Hence prove that if a sequence (x_n) is convergent then it is Cauchy. (5 marks)

(b) Using the definition of limit of a function, prove that

(i) $\lim_{n\to\infty}\left(\frac{(-1)^n}{n+5}\right) = 0$ (3 marks)

(ii) $\lim_{x\to 2}(x^3 + x - 10) = 0$ (4 marks)

(c) Using the function $f(x) = x^{\frac{1}{3}}$ at the point $x = 0$, show that the property of continuity does not necessarily imply differentiability. (8 marks)
