CHUKA UNIVERSITY

SUPPLEMENTARY/ SPECIAL EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF COMMERCE

MATH 206: INTRODUCTION TO REAL ANALYSIS

STREAMS: Y2S2 **TIME: 2 HOURS**

DAY/DATE: TUESDAY 02/02/2021 **8.30 AM – 10.30 AM**

INSTRUCTIONS:

- Answer question **ALL** the questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write on the question paper
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

(a) Using the concepts of limiting points and neighborhoods, explain whether the following sets are open, closed or none.

(i) $S = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots.\right\}$

(ii) $S = \{(-1)^n : n \in \mathbf{N} : \}$ (4marks)

(b) Let a and b be non-negative real numbers. Prove that

(i) there exist always a non-negative real number a^{-1} (3 marks)

(ii) $a < b$ if and only if $a^2 < b^2$ (4 marks)

(c) Given the set $A = \{x \in \mathbb{R} : a \leq x < b\}$. Determine if possible, the lower boundary, infimum, upper boundary and supremum of the set. (4 marks)

(d) Prove that if x and y are positive real numbers then

(i) x + y is also positive (2 marks)

(ii) x<y $\Rightarrow \frac{1}{y} < \frac{1}{x}$ (3 marks)

(e) Define a countable set. Hence illustrate that Illustrate that rational numbers between $[0, 1]$ is countable the set of real numbers $\mathbb{R}$ is uncountable. (5 marks)

(f) Using the first principle of the definition of the derivative of a function f at x find the derivative of the function $f(x) = \sqrt{x}$ (5 marks)

QUESTION TWO: (20 MARKS)

(a) Given that $A \subseteq \mathbb{R}$, show that A is open iff $A = A^0$ (5 marks)

(b) Prove that a limit of function exists then that limit is unique (5 marks)

(c) State the conditions to be satisfied for a function to be continuous at a point $x = c$. Hence show that the functions $f(x) = |x - 2|$ is contionus at the point x=2 but not differentiable at the same point. (10 marks)

QUESTION THREE: (20 MARKS)

(a) Define a Cauchy sequence (x_n) in R. Hence prove that if a sequence (x_n) is convergent then it is Cauchy. (6 marks)

(b) Find the limit superior and limit inferior of the sequence

$X_n = \left(1 + \frac{n}{n+1} + cos\ \frac{n\pi}{2}\right) : n \in \mathbf{N}$ (6 marks)

(c) Hence show that the function $f(x) = 2x$ is Riemann Integrable on the interval [0,1]

(8 marks)
