CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF

BACHELOR OF SCIENCE DEGREE IN

MATH 121: CALCULUS I

STREAMS:

TIME: 2 HOURS

DAY/DATE: WEDNESDAY 31/3/2021 **8.30 AM – 10.30 AM**

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** Questions

QUESTION ONE: 30 MARKS (COMPULSORY]

a) Find the derivatives of the following functions.

i) $f(x) = cosx\ sinx$ [2 Marks]

ii) $3x^2y$-4y$-2x + 1 = 0$ [3 Marks]

iii) $y = e^{\sqrt{x^2}+1}$ [2 Marks]

b) Find d_y^2 / dx^2given that

y^3-x^2= 4 [3 Marks]

c) Evaluate

lim x⋯→1 $\left[\frac{\sqrt{3+x}\ -\ \sqrt{5-x}}{x^3\ -1}\right]$
x⟶ 1

d) State without proof the Rolle's theorem. [2 Marks]

f) Find the coordinates of the points on the curve $y = x^3 - 6x^2 + 12x + 2$ at which the tangent is parallel to the line $y = 3x$ [3 Marks]

g) Find the gradient function of the given expression $f(x) = x^3 + 5$. Using first principles. Hence evaluate gradient at the point (2,13). [3 Marks]

h) Use a differential to estimate the change in $f(x)=x^{2/5}$. If x is decreased from 1 to $^9/_{10}$.

[3 Marks]

i) Find $\frac{dy}{dx}$ if $y = Sin^{-1}3x - Cos^{-1}3x$ [3 Marks]

j) Analyse the continuity of the function.

$$f(x) = \begin{cases} x & 0 \leq x < 1 \\ \frac{1}{2}x & 1 \leq x < 2 \end{cases}$$

[3 Marks]

QUESTION TWO (20 MARKS)

a) *Find y' if* $y = \frac{3x^2 - x + 2}{4x^2 + 5}$ [3 Marks]

b) Determine the discontinuities of function. [6 Marks]

$$F(x) \begin{cases} x^3, & x \leq -1 \\ x^2 - 2, & -1 < x < 0 \\ 3 - x, & 0 \leq x < 2 \\ \frac{4x-1}{x-1}, & 2 \leq x < 4 \\ {}^{15}/_{7-x}, & 4 < x < 7 \\ 5x + 2, & 7 \leq x \end{cases}$$

c) Show that $^d/_{dx}\, Sin^{-1}u = \frac{1}{\sqrt{1-u^2}} \frac{d}{dx} u$ [3 Marks]

d) Find the derivative of the function. [3 Marks]

$f(x) = x^x$ Where $x > 0$

e) Evaluate

$$\lim_{x \to 0} \left(\frac{1-cost}{t}\right)$$

[3 Marks]

g) State the mean value theorem. [2 Marks]

QUESTION THREE (20 MARKS)

a) Differentiate x^2e^x with respect to x

[3 Marks]

b) If x = a $Sec\,^2_\theta$, and y = a $tan^3\theta$. Find $\frac{d^3y}{dx^3}$ [6 Marks]

c) If f(x) = 4x /(x^2+9)

i) Determine the horizontal asymptotes. [2 Marks]

ii) Find the relative maxima and minima and the points of inflection. [3 Marks]

iii) Discuss the concavity and sketch the graph. [3 Marks]

d) Assuming that equation

y^4+ 3y -4x^3= 5x + 1

Determine implicitly, a differentiable function f such that Y= f (x), find its derivative.

[3 Marks]

QUESTION FOUR (20 MARKS)

a) Prove that $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)f^1(x)-f(x)g^1(x)}{[\ g(x)\ ]^2}$

Where g(x) ≠O. Using first principle approach. [4 Marks]

b) Find the limit of the function.

$$\lim_{x \to 0} \frac{x^2}{secx-1}$$

c) Evaluate

$\frac{d}{dt} \left[t^3 - \frac{1}{t^2-1} \right]$

d) Find the local extrema of the function.

$f(x) = 2Sin\, x + Cos2x$

On the interval $[0, 2\pi]$ [5 Marks]

f) Use first principle to differentiate

f(x) = $\frac{1}{x}$ x$\neq$ 0 [2 Marks]

QUESTION FIVE (20 MARKS)

a) Find y^1 if y = $3^{\sqrt{x}}$ [2 Marks]

b) If f(x) = $4x^2$- 5x + 8-$\frac{3}{x}$. Find the four derivatives of f(x) [4 Marks]

c) Use Newton's Method to approximate the largest positive root of the equation.

x^3-3x +1 =0 to four decimal places using x_1= 1.5 [4 Marks]

d) Verify the intermediate value theorem if f(x) =$\sqrt{x+1}$ and the interval is $[3, 24]$ [4 Marks]

e) Sketch the intersecting graphs of the equations $2x^2$+ y^2= 6 and $y^2 = 4x$ and show that they are orthogonal. [6 Marks]

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