CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF SCIENCE (ACTURIAL SCIENCE)

MATH 306: FUNDAMENTALS TO REAL ANALYSIS

STREAMS: `` As above``

TIME: 2HRS

DAY/DATE: ……………………. ………………………..

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** Questions

QUESTION ONE (30 MARKS)

a) Define the following terms as used in analysis
   (i) A neighborhood of a point $x_0 \in \mathbb{R}$ (1 mark)
   (ii) An interior point of a subset $A \subset \mathbb{R}$ (1 mark)
   (iii) A limit point x of a subset A of ℝ. (1 mark)

b) Distinquish an open and closed subset of ℝ. Hence determine whether the sets $A = \left\{\frac{1}{n} : n \in N\right\}$is open or closed. (4 marks)

c) Distinguish and odd and even integer number m. Hence prove that an integer number m is even iff m^2 is even. (5 marks)

d) State without proof the Completeness axiom for real numbers. (2 marks)

e) Use precise definition to prove that $\lim_{x\to 3}(t^3 + 3t) = 36$ (3 marks)

f) Find if the following sets are bounded or not and if bounded find the $sups$ and $infs$

(i) $S_1 = \{x \in \mathbb{R}: a \leq x < b\}$ (2marks)

(ii) $S_2 = \left\{1 + (-1)^n \frac{1}{n} : n \in \mathbf{N}: \right\}$ (2marks)

(iii) $S_2 = \{1 + (-1)^n . n : n \in \mathbf{N}: \}$ (2marks)

g) (i) State the Intermediate Value Theorem (2 marks)
(ii) Hence use it to show that the $f(x) = x^3 - 2x^2 + 2x - 4$ has a zero in the interval $[0, 3]$ (2 marks)

h) Discuss the following concepts as used in analysis
i. A partion of a closed interval $[a, b]$ (1 mark)
ii. The Riemanns' upper sum and lower sum of the function f (2 marks)

QUESTION TWO (20 MARKS)

a) Prove that between any two rational numbers, there is an irrational number (4 marks)

b) Given that $x, y \in \mathbb{R}$. Then,
(i)Show that if x is positive then $-x$ is negative (2 marks)

(ii) $x < y \Rightarrow \frac{1}{y} < \frac{1}{x}$ (2 marks)

(iii) $x < y$ iff $x^2 < y^2$ (2 marks)

c) (i) Given that $A \subseteq \mathbb{R}$, show that A is open if and only if $A = A^0$ (5 marks)
(ii) Let $\bar{A}$ denote the closure of a subset $A \subset \mathbb{R}$. Prove that $A = \bar{A}$ if and only if A is closed. (5 marks)

QUESTION THREE (20 MARKS)

a) Define a Cauchy sequence (x_n) in R. Hence prove that if a sequence (x_n) is convergent then it is Cauchy. (6 marks)

b) Prove that every convergent sequence is bounded. Using an appropriate counter example show that the converse of this is not necessarily true (7 marks)

c) Find the limit superior and limit inferior of the sequence
$X_n = \left(sin\ \frac{n\pi}{2} + \frac{(-1)^n}{n}\right) : n \in \mathbf{N}$ (7 marks)

QUESTION FOUR (20 MARKS)

a) State the conditions to be satisfied for a function to be continuous at a point $x = c$. Hence show that the functions $f(x) = |x - 2|$ is contionus at the point x=2 but not differentiable at the same point. (10 marks)

b) Describe the Riemann Integrable function f on the interval $[a, b]$. Hence show that the function $f(x) = 3x$ is Riemann Integrable in $[0,1]$ (10 marks)

QUESTION FIVE (20 MARKS)

a. (i) Show that the set of integers is countable (4 marks)
   (ii) Prove that an infinite subset of a countable set is countable. Hence or otherwise show that the set of even numbers is countable. (6 marks)

b. Using appropriate examples, explain the three different types of discontinuities of a function (6 marks)

c. Using the first principle find the derivative of $y = \frac{3}{x}$ (4 marks)