SECOND YEAR **SECOND** SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

MATH 202 – FUNDAMENTALS OF LINEAR ALGEBRA

DURATION: 2 HOURS

DATE:

TIME:

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE

a) Compute the determinant (2 marks)

$$C = \begin{bmatrix} \frac{1}{2} & -1 & -\frac{1}{3} \\ \frac{3}{4} & \frac{1}{2} & -1 \\ 1 & -4 & 1 \end{bmatrix}.$$

b) Let u=2i-3j+4k , v=3i+j-2k , w=i+5j+3k

Find u × v and u ×w (6 marks)

c)Determine whether or not the vectors u= (1,1,2), w= (4,5,5) in R^3 are linearly dependent (4 marks)

d) Reduce the following matrix to echelon form (3 marks)

$$A = \begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -4 & 3 \end{bmatrix}$$

e) show that the below matrices are inverses (4marks)

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix} \text{ and } B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$$

f)Define what is a vector subspace (2 marks)

g)

$$\text{Let } A = \begin{bmatrix} 1 & -2 & 3 & 1 & 2 \\ 1 & 1 & 4 & -1 & 3 \\ 2 & 5 & 9 & -2 & 8 \end{bmatrix}.$$

Use Gauss–Jordan to find the row canonical form of A (9 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO

a)Express u=(1,-2,5) in R^3 as a linear combination of the vectors(5 marks)

u_1 = (1,1,1) , u_2 =(1,2,3) , u_3 =(2,-1,1)

b) Find all eigenvalues and corresponding eigenvectors of A (6 marks)

$$A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}.$$

c)

Let $A = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$, a real symmetric matrix.

Find an orthogonal matrix P such that $P^{-1}AP$ is diagonal (9 marks)

QUESTION THREE

a) Evaluate the following system using Gaussian elimination(3 marks)

$$\begin{aligned} x + 2y - 4z &= -4 \\ 2x + 5y - 9z &= -10 \\ 3x - 2y + 3z &= 11 \end{aligned}$$

b) Find solution using revised simplex method (17 marks)

MAX $Z=3X_1 +5X_2$

Subject to

$X_1 \leq 4$

$X_2 \leq 6$

$3X_1 +2X_2 \leq 18$

And $X_1, X_2 \geq 0$

QUESTION FOUR

a) Suppose u= (1,2,3,4) and v=(6,k,-8,2).Find k so that u and v are orthogonal. (3 marks)

b)Suppose u=(1,-2,3) and v=(2,4,5). Find the distance, angle and projection (6 marks)

c)Find the characteristic polynomial of the below matrix (6 marks)

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & 3 & 9 \end{bmatrix}.$$

d) Find the quadratic form q(x) that corresponds to the symmetric matrix (5 marks)

$$B = \begin{bmatrix} 4 & -5 & 7 \\ -5 & -6 & 8 \\ 7 & 8 & -9 \end{bmatrix}$$

QUESTION FIVE

a)Find the inverse (3 marks)

$$A = \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}$$

b) Determine if A is an orthogonal matrix(5 mark)

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

c)

Let $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$

$f(x) = 2x^3 - 4x + 5$ and $g(x) = x^2 + 2x + 11.$

Find

i. A^2
ii. A^3
iii. f(A)
iv. g(A)

(12 marks)