CHUKA   **UNIVERSITY**

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF EDUCATION SCIENCE/ARTS, BACHELORS OF SCIENCE CHEMISTRY/INDUSTRIAL CHEMISTRY, BACHELORS OF ARTS(MATHS-ECONS), BACHELORS OF SCIENCE(ECON STATS)

MATH 201: LINEAR ALGEBRA I

STREAMS: ````as above```` Y2S2   **TIME: 2HRS**

DAY/DATE: …………………….   ………………………..

INSTRUCTIONS:

- Answer question **ONE** and **TWO** other questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

a) Consider the system in unknowns $x \; and \; y$

$$ax + 9y = b$$
$$2x + ay = 4$$

Find which values of (a, b) that give the three types of solutions to the system   **(4 marks)**

b) Evaluate the WROSKIAN $W(\sin x, \sin 2x, \cos x, \frac{\pi}{4})$ **(4 marks)**

c) Distinguish the Kernel and Range of a transformation T. Hence prove that if $T: U \to V$ is a linear transformation, then the kernel of T is a subspace of U.   **(5 marks)**

d) Show that the subset $W = \{(x, y) : x + y = 0, x, y \in R^2\}$ is a subspace of R^2 **(3 marks)**

e) Determine if $T: R^3 \rightarrow R^2$ defined as $T(x_1, x_2, x_3) = (x_1 + x_3, \ 2x_2, -x_3)$ is a linear transformation. (4 marks)

f) Verify whether or not the vector (1,2,-1) is a linear combination of the vectors $\{(1,1,-1),(2,2,1),(-1,-1,2)\}$. Can these vectors form a basis for $\mathbb{R}^3$ (5 marks)

g) Given the following basis for $\mathbb{R}^3$ $B = \{(1,0,0),(0,1,0),(0,0,1)\}$ and $B' = \{(1,0,1,),(2,1,2),(1,2,2)\}$ find a transition matrix from B to B' (5marks)

QUESTION TWO: (20 MARKS

a) For which values of a does the below system has
    (i)   No solution

    (ii)   Unique solution

    (iii)   Infinitely many solutions.

$$ax_1 + x_2 + x_3 = -1$$

$$x_1 + ax_2 + x_3 = 4$$

$$x_1 + x_2 + ax_3 = 1$$

(8 marks)

b) (i)Using of row reduction method, find the inverse for the matrix A = $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 6 \\ 1 & 1 & 4 \end{bmatrix}$

(ii) Hence or otherwise find all the solutions to the system $A^2 x = b$ where b is the vector (0,1,1)

(8 marks)

(c) For a matrix $A_{(m \ x \ n)}$, prove that

    i)   If A is invertible, then $A\underline{x} = \underline{b}$ has a unique solution for any b   (2 marks)
    ii)   If A is row equivalent to an identity matrix I_n, then A is invertible   (2 marks)

QUESTION THREE: (20 MARKS)
    a) Find the inverse of the following matrix by first getting the adjoint

$$\begin{pmatrix} 2 & 1 & -2 \\ 3 & 2 & 2 \\ 5 & 4 & 3 \end{pmatrix}$$

Hence or otherwise, solve the following system of linear equations

$$2x_1 + x_2 - 2x_3 = 10$$

$$3x_1 + 2x_2 + 2x_3 = 1$$

$$5x_1 + 4x_2 + 3x_3 = 4$$

(7 marks)

b) Let $F: \mathbb{R}^5 \to \mathbb{R}^4$ be a linear mapping defined by
$F(x, y, z, w, t) = (x + 2y + w - t, 2x - y + 3z + t, -x - 2z + t, 2w + 8t)$. Find

    i.   The basis and dimension of the kernel of F

    ii.   A basis and dimension of the image of F

    iii.   Using the parts i) and ii) above, verify the dimension theorem   (7 marks)

c) Show that the vectors $\{1, x - 1, x^2 - 1\}$ is a basis for the vector space $P^2(x) =$ the space of all polynomials in x of degree less or equal to 2   (6 marks)

QUESTION FOUR: (20 MARKS)

a) Solve the following system of equations of the planes by use of Gauss Jordan elimination method

$$2x_1 + x_2 + x_3 = 1$$

$$-x_1 + 2x_2 - 3x_3 = 3$$

$$x_1 + 3x_2 - 2x_3 = 4$$

Hence give the geometrical interpretation of the solution of the planes   (5 marks)

b) (i) Prove that if S is a subset of a vector space V, then the set spanned generated by S is a subspace of V.   (3 marks)

ii) Let $V = \mathbb{R}^3$ and S = {(2, 3, 5), (1, 2, 4), (-2, 2, 3)}. Determine if (10, 1, 4) $\in span$ S   (4 marks)

c) Let $T : \mathbb{R}^{\wedge}3^2 \to R^2$ be a linear transformation defined by $T(x, y) = (x - 2y, -x + 3y)$

    i.   Find the matrix of T relative to the basis $B = \{(1,0), (1,1)\}$ (3 marks)

    ii.   Find the matrix of T relative to the basis $B' = \{(1,-1), (1,2)\}$ (3 marks)

    iii.   Find the transition matrix P from the basis B to the basis B' and verify the relation $P^{-1}[T]_B P = [T]_{B'}$ (3 marks)

QUESTION FIVE: (20 MARKS)

a) Use Cramer's method to solve the system of equation

    x + y -2z = -3

    w + 2x - y = 2

$$2w + 4x + y - 3z = -2$$

$$w - 2x - 7y - z = 5 \qquad \text{(8 marks)}$$

b) Find the basis and dimension of the solution space for the equations

$$2x_1 + 2x_2 - x_3 + x_5 = 0$$

$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$

$$x_1 + x_2 - 2x_3 - x_5 = 0$$
$$x_3 + x_4 + x_5 = 0 \qquad \text{(7 marks)}$$

c) Prove that any two bases defined on the same vector space have the same number of vectors.

(5 marks)