CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF MASTERS OF SCIENCE IN APPLIED STATISTICS

MATH 848: STOCHASTIC PROCESSES

STREAMS: MSc (App Stat) **TIME: 2 HOURS**

DAY/DATE:

INSTRUCTIONS:

- **Answer ANY THREE Questions**.

QUESTION ONE [20 MARKS]

a) Let X and Y be independent Poisson random variables with parameters $\lambda \wedge \mu$ respectively. Given that $Z = X + Y$; Find the:
   i) p.g.f of X and p.g.f of Y
   ii) p.g.f of Z (5 marks)

b) A credit union classifies automobile loans into one of four categories: the loan has been paid in full (F), the account is in good standing (G) with all payments up to date, the account is in arrears (A) with one or more missing payments, or the account has been classified as bad debt (B) and sold to a collection agency. Past records indicate that each month 10% of the accounts in good standing pay the loan in full, 80% remain in good standing and 10% become in arrears. Furthermore, 10% of the accounts in arrears are paid in full, 40% become accounts in good standing, 40% remain in arrears, and 10% are classified as bad debts.
   i) In the long run, what percentage of the accounts in arrears will pay their loan in full?
   ii) In the long run, what percentage of the accounts in good standing will become bad debts?
   iii) What is the average number of months an account in arrears will remain in this system before it is either paid in full or classified as a bad debt?(10 marks)

c) A mail-order company classifies its customers as preferred, standard, or infrequent, depending on the number of orders placed in a year. Past records indicate that each year 5% of the preferred customers are reclassified as standard and 12% as infrequent; 5% of the standard customers are reclassified as preferred and 5% as infrequent; and 9% of the infrequent customers are reclassified as preferred and 10% as standard. Assuming that these percentages remain valid.
   i) Illustrate this process with a transition diagram
   ii) What percentage of customers can the company expect to have in each category in the long run? (5 marks)

QUESTION TWO [20 MARKS]

a) Consider a birth-death process with $\mu=0$ and $\lambda=0$ with parameter $k\geq 0$ as the immigration rate. Given the initial conditions as

$$p_n(0)=\begin{cases}1, n=0\\0, n\neq 0\end{cases}$$

Find:

i) $p_n(t)$

ii) E(n) and var(n) (12 marks)

b) Prove that the random process $x(t)=A\cos(wt+e)$ is stationary if it is assumed that A and w are constants and x is a uniformly distributed random variable on the interval ($0, 2\pi$). (8 marks)

QUESTION THREE [20 MARKS]

a) Define the following terms:

i) Generating function

ii) Random process

iii) Stochastic process

iv) Regular markov chain

v) Poisson process (5 marks)

b) Classify the states of the following transition probability matrix

$$\begin{bmatrix} 0.8 & 0.2 & 0 & 0 & . & . \\ 0.8 & 0 & 0.2 & 0 & . & . \\ 0.8 & 0 & 0 & 0.2 & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0.8 & . & . & . & . & 0.2 \end{bmatrix}$$

(10 marks)

c) Use the matrix below to answer the questions that follow

$$P=\begin{bmatrix} 1. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. \\ .1 & .1 & .7 & .1 \\ .3 & .1 & .4 & .2 \end{bmatrix}, S_0=[.1 \quad .2 \quad .3 \quad .4]$$

Find:

i) The limiting matrix for P

ii) The long-run probability of going from each non-absorbing state to each absorbing state.

iii) The average number of trials needed to go from each non-absorbing state to each absorbing state. (5 marks)

QUESTION FOUR [20 MARKS]

a) Let Y be the family size or the number of offsprings distribution.
Z_n be the size of the population at time n or the size of generation n, where n = 0, 1, 2, …
Let the $E(Y)=\mu$ and $Var(Y)=\sigma^2$, the mean and variance of the number of offspring of a single individual. Suppose that $\{Z_0, Z_1, Z_2, \ldots\}$ is a branching process with $Z_0=1$ (starting with a single individual).
Then proof that:

1) $E(Z¿¿n)=\mu^n¿$

2) $Var(Z_n)=\begin{cases} n\sigma^2, if\ \mu=1 \\ \sigma^2\mu^{n-1}\left(\frac{1-\mu^n}{1-\mu}\right), if\ \mu\neq 1 \end{cases}$

3) Using (1) and (2) above and given that the family size $Y\sim Geometric(p=0.3)$ find the:
   i) Expected population size by generation n = 10, [$E(Z¿¿10)¿$]
   ii) Variance of the population by generation n = 10, [$Var(Z_{10})¿$ (12 marks)

b) Let Z_t be a random variable denoting the position at time t of a moving particle

(t = 0, 1, 2,…). Initially, the particle is at the origin, $Z_0=0$. At time t,

$Z_t=Z_{t-1}+X_t$, where X_t, the jump at the t^th step, is such that the random variables $\{X_1, X_2, \ldots\}$ are mutually independent and all have the distribution.

$P(X_t=1)=P(X_t=-1)=\frac{1}{2}$, (t = 1, 2, …).

With $Z_0=0$, $Z_t=X_1+X_2+\ldots+X_t$ whose distribution is closely related to the binomial distribution. The probability that the particle visits the origin at time t = 2m is

$$P(Z_{2m}=0)=\binom{2m}{m}2^{-2m}=\frac{(2m)!}{m!m!}2^{-2m}$$

i) What is the probability that the particle returns to the origin?
ii) What is the expected waiting time until this happens? (8 marks)