CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTERS OF SCIENCE AND APPLIED STATISTICS

MATH 845: STATISTICAL INFERENCE

STREAMS: MSC **TIME: 3 HOURS**

DAY/DATE: THURSDAY 07/04/2021 **2.30 P.M. – 5.30 P.M.**

INSTRUCTIONS:

- **Answer ANY THREE Questions**.

QUESTION ONE [20 MARKS]

a) Define the terms given below illustrating with an example
   i) Ancilliarity
   ii) Asymptotically efficient estimator
   iii) Invariant test (6 marks)

b) Suppose that $x_1, x_2, x_3, \ldots, x_n \; i.i.d\, Bernoulli(p)$
   i) Find the maximum likelihood estimator (MLE) for p (3 marks)
   ii) Find the standard error estimate for p using the Fisher Information. (6 marks)
   iii) Taking $\alpha = 0.05$, construct an asymptotic confidence interval for p. (5 marks)

QUESTION TWO [20 MARKS]

a) Consider a random sample of n independent random variables $Y_1, Y_2, Y_3, \ldots, Y_n$ be i. i. d B (1, θ) with p.m.f.

$$f(x,\theta) = \begin{cases} \theta^x (1-\theta)^{1-x}, x = 0,1 \\ 0, otherwise \end{cases}$$

Find a minimal sufficient statistic for θ. (10 marks)

b) Let $x_1, x_2, x_3, \ldots, x_n \; i.i.d\, N(\mu, \sigma^2)$ where μ and σ^2 are unknown.

Show that $T=\left(\sum_{i=1}^{n} x_i, \sum_{i=1}^{k} x_i^2\right)$ is:

i) A 2-parameter exponential family

ii) A complete and sufficient statistic for $\theta=(\mu,\sigma^2)$ (10 marks)

QUESTION THREE [20 MARKS]

a) Let $y_1, y_2, y_3, \ldots, y_n$ be i. i. d Bernoulli (1, p) random variables with p.m.f.

$$f(x,p)=\begin{cases} p(1-p)^{1-x}, x=0,1 \\ 0, otherwise \end{cases}$$

Find the Uniformly Most Powerful size α test for $H_0: p=p_0$ against $H_1: p=p_1$ where $p_1 > p_0$ (10 marks)

b) Let $z_1, z_2, z_3, \ldots, z_n$ $i.i.d\, N(\mu,\sigma^2)$ random variables. Find the Likelihood Ratio Test (LRT) for $H_0: \mu=\mu_0$ against $H_0: \mu \neq \mu_0$ and in a case where σ^2 is unspecified. (10 marks)

QUESTION FOUR [20 MARKS]

Let $x_1, x_2, x_3, \ldots, x_n$ $i.i.d\, N(\mu,\sigma^2)$

a) Show that $\hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\overline{x})^2$ is asymptotically biased estimator of σ^2. How would you fix the bias? (5 marks)

b) Show that the sample mean is a UMVUE for μ (7 marks)

c) Obtain the asymptotically efficient MLE for unbiased estimator of σ^2 (8 marks)
