CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

MATH 842: MEASURE AND PROBABILITY THEORY

STREAMS: Msc APPLIED STATISTICS PART TIME

TIME: 3 HOURS

DAY/DATE : WEDNESDAY 6 /10/ 2021 **2.30 PM – 5.30 PM**

INSTRUCTIONS:

- Answer Any THREE Questions

QUESTION ONE (20MARKS)

(a) State and explain the eight set operations given that *CCP* (Ω) is a collection of subjects Ω .

[8 Marks]

(b) State the four definitions related to weak convergence of probability measures given that ¿) is a probability distribution function and F is also a probability distribution which is not necessarily proper. [8 Marks]

(c) Define the following terms as described under random elements of metric spaces.

(i) Random variable [2 Marks]

(ii) Random vector [2 Marks]

QUESTION TWO (20 MARKS)

a) Let =R and that P is a probability measure on R. Show $F(x)=P(\in \infty, x$¿$), x \in R$ is.Ω =R

(i) Right continuous

(ii) Monotone Increasing

(iii) Has limits at $\pm \infty$ [10 Marks]

(b) The Faton lemma states that $if\ x_n \geq 0\ then\ E(\lim Inf\ x_n) \leq \lim Inf\ E(x_n)$.

$n \longrightarrow \infty \qquad n \longrightarrow \infty$

If there exists $Z \epsilon L_1$ and $x_n \geq Z$, then prove that $E\left(\lim_{n \to \infty} x_n\right) \leq \lim_{n \to \infty} info\ E(x_n)$

[10 Marks]

QUESTION THREE (20 MARKS)

a) If [x_n¿are independent random variables with tail ϭ-field T, then A ∈ T implies $P(A)=0$ or 1 so that the tail $\sigma-$ field T is almost trivial (Hint; kolmogoro zero-one law)

[15 Marks]

b) Given that A = { 9, 8, 10, 11, 12} and B= { 6, 7, 9, 11, 12, 14, 18}

Find the following indicator functions

(i) $I_A 11$

(ii) $I_B 10$

(iii) $I_{AB} 12$

(iv) $I_{A \cup B} 8$

(v) $I_{B^c} 9$ [5 Marks]

QUESTION FOUR (20 MARKS)

State the properties of the expectation operator E [20 Marks]

QUESTION 5 (20 MARKS)

(a) Suppose that $\{x_n, \eta \geq 1, x\}$ are random variables on a probability space ($\Omega \beta \rho$) if $x_n \to x\, a.s$ then prove $x_n \overset{P}{\to} x$. [5 Marks]

(b) Let $\{x_n, n, \geq 1\}$ be iid random variables with $E(x_n)=\mu$ and ¿ $Var\ (x_n$¿$=\sigma^2$.
Suppose N is a random variable with $N(0,1)$ distribution if $S_n = x_1 + x_2 + \ldots + x_n$ Then

prove that $\frac{S_n - n\mu}{\frac{\sigma}{\sqrt{n}}} \Rightarrow N$ [15 Marks]

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