CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF MASTER OF SCIENCE

MATH 829: METHODS OF FLUID MECHANICS II

STREAMS: MSC **TIME: 3 HOURS**

DAY/DATE: TUESDAY 06/04/2021 **8.30 A.M. – 11.30 A.M.**

INSTRUCTIONS: Answer any THREE questions

QUESTION ONE (20 MARKS)

(a) (i) State the basic idea in using the finite difference techniques [2 marks]

(ii) Outline the three steps followed in in solving a problem using a finite difference method. [3 marks]

(iii) Using mesh schematic diagrams explain the differences between explicit and implicit methods of solving pde [6 marks]

(b) Solve the boundary value problem using the difference Scheni

$$\frac{y_{i-1}-2y_i+y_{i+1}}{h^2}+\frac{y_{i+1}-y_{i-1}}{2n}+x_i=0$$

$y''+y'+x=0$ with boundary condition $y(o)=0, y(1)=0$ [9 marks]

QUESTION TWO (20 MARKS)

(a) Consider the one dimension equation

$$\frac{d^2u}{dt^2}=c^2\frac{d^2u}{dx^2}, t>0, 0<x<1$$

Using central difference approximation at a mesh point (ih, jk) set up the difference formulation using

(i) Implicit method I [3 marks]

(ii) Implicit method II [3 marks]

(b) Consider the wave equation

$\frac{d^2u}{dt^2} = c^2 \frac{d^2u}{dx^2}$ with

$u(o,t) = 0, u(1,t) = 0, t > 0$ and

$u(x,o) = 4x^2, \left(\frac{du}{dt}\right) = 0 \; 0 \le x$

Using central differences and the explicit formula find the value of u for $x = 0, 0.2, 0.4 \wedge t = 0, 0.1 \ldots 0.5$ when c=1 [15 marks]

QUESTION THREE (20 MARKS)

(a) (i) Explain the meaning of a well posed mathematical problem [3 marks]

(ii) Using examples state and differentiate between the 3 types of boundary conditions used in solving partial differential equations [6 marks]

(b) Solve by the Crank-Nickolson method

$U_{xx} = U_t$ Subject to

$U(x,o) = 0, u(o,t) = 0$

$U(1,t) = t$ for 2 time steps, taking $h = ¼$ [11 marks]

QUESTION FOUR (20 MARKS)

(a) (i) Write an explicit finite difference scheme for the

$\frac{du}{dt} = \frac{d^2u}{dx^2}$ for $0 < x < 1, t > o$

Given that

$u(o,t) = \frac{du}{dt}(1,t) = 0, u(x,0) = f(x)$ and $\left(\frac{du}{dt}\right)_{x\,10} = g(x) 0 < x < 1)$ [3 marks]

(iii) Write an implicit finite difference scheme for the problem

$U_t = U_{xx}$ for $0 < x < 1, t > 0$

Given that

$$U(o,t)=U_x(1,t)=0, U(x,o)=f(x) \text{ and } \left(\frac{du}{dt}\right)_{x,o}=g(x), 0<x<1)$$

(b) Discuss the classification of the general linear partial differential equation of the form.

[6 marks]

$$AU_{xx}+BU_{xy}+CU_{yy}+DU_x+EU_y+FU=0$$

(c) Solve the system of linear equations using Jacobi method with $x^{\circ}=(1,1,1)^T$ and three iterations

$$5x_1+x_2-x_3=4$$

$$x_1+4x_2-2=15$$

$$x_1-2x_2+5x_3=12$$

[8 marks]
