**CHUKA UNIVERSITY**

**UNIVERSITY EXAMINATIONS 2020/2021**

**EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS**

**MATH829: METHODS OF FLUID MECHANICS II** **TIME: 3HOURS**

**DAY/DATE: APRIL 2021**

**INSTRUCTIONS**

Answer any **THREE** Questions

**QUESTION ONE (20MARKS)**

**a.** i. State the basic idea in using the finite difference techniques for solving differential equations

(2Marks)

ii. Outline the three steps followed in solving differential equations using finite difference method (3Marks)

iii. Using mesh schematic diagrams, explain the differences between Explicit and Implicit methods of solving Partial differential Equations (6Marks)

b. Solve the boundary value problem using the difference scheme (11Marks)

$$\frac{y_{i-1}-2y_i+y_{i+1}}{h^2}-\frac{y_{i+1}-y_{i-1}}{2h}+x_i=0$$

**QUESTION TWO (20MARKS)**

**a.** Consider the one dimension equation

$$\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}\ \ for\ 0<x<1\ ,\ t>0$$

$$Given\ that$$

$$u(0,t)=\frac{\partial u}{\partial t}(1,t)=0,\ u(x,0)=f(x)\ and\ \left(\frac{\partial u}{\partial t}\right)_{(x\ ,\ 0)}=g(x)\ ,\ 0<x<1$$

Using the central difference approximation at a mesh point $(\hat{i}h\ ,\ \hat{j}k)$, set up the difference formulation using

i. Implicit Method I (3Marks)

ii. Implicit Metho II (3Marks)

b. Consider the wave equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad with$$

$$u(0,t) = 0 \; , \; u(1,t) = 0 \; , \; t > 0$$

$$and$$

$$u(0,t) = 4x^2 \, , \frac{\partial u}{\partial t} = 0 \, , \left(\frac{\partial u}{\partial t}\right) = 0 , \; 0 \leq x$$

Using Central differences and the explicit formula find the values of $u \; for \; x = 0, \; 0.2, \; 0.4$ and

$t = 0, \; 0.1 \, , 0.2, \; 0.3 \; , 0.4 \; , \; 0.5$ when $c = 1$

(14Marks)

**QUESTION THREE (20MARKS)**

a. i. Explain the meaning of a well posed Mathematical problem (3Marks)

ii. Using examples state and differentiate between the 3 types of boundary conditions used in solving Partial differential equations (6Marks)

b. Use the Crank- Nickolson method to solve (11Marks)

$u_{xx} = u_t$ Subject to:

$$u(x,t) = 0 \; , \; u(0,t) = 0 \; , \; t > 0$$

$$and$$

$$u(1,t) = t \;\; for \; 2 \; time \; step \; with \; h = \frac{1}{4}$$

**QUESTION FOUR (20MARKS)**

a. i. Write an explicit difference scheme for solving the boundary value problem (6Marks)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad for \; 0 < x < 1 \; , \; t > 0$$

$$Given \; that$$

$$u(0,t) = \frac{\partial u}{\partial t}(1,t) = 0, \; u(x,0) = f(x) \; and \; \left(\frac{\partial u}{\partial t}\right)_{(x\,,\,0)} = g(x) \; , \; 0 < x < 1$$

ii. Write an explicit difference scheme for solving the boundary value problem (6Marks)

$$u_t = u_{xx} \; for \; 0 < x < 1 \; , t > 0$$

*Given that*

$$u(0,t) = \frac{\partial u}{\partial t}(1,t) = 0, u(x,0) = f(x) \; and \; \left(\frac{\partial u}{\partial t}\right)_{(x\,,\,0)} = g(x) \; , 0 < x < 1$$

b. Discuss the classification of the general linear Partial Differential euation

$$Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = 0$$

(6Marks)

c. Solve the system of linear equations using the Jacobi method with $x^0 = (1\,,1\,,1)^T$ and three iterations (8Marks)

$$5x_1 + x_2 - x_3 = 4$$
$$x_1 + 4x_2 + 2x_3 = 15$$
$$x_1 - 2x_2 + 5x_3 = 12$$