CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

MATH 822: ORDINARY DIFFERENTIAL EQUATIONS II

STREAMS: MSC (MATH) **TIME: 3 HOURS**

DAY/DATE: TUESDAY 06/04/2021 **2.30 P.M. – 5.30 P.M.**

INSTRUCTIONS: Answer any THREE Questions

QUESTION ONE (20MARKS)

a. Explain the meaning of the following Mathematical terms (8 marks)

- (i) Eigen Function
- (ii) Eigenvalue problem
- (iii) Sturm Liouville Problem
- (iv) Boundary Value Problem

b. For the Boundary Value Problem $\frac{d^2y}{dx^2}+\lambda y=0;\ y(0)=y(c)=0$, find

- i. Eigenvalues (6 marks)
- ii. Eigen functions (6 marks)

QUESTION TWO (20MARKS)

a. State the following

- i. The condition for a Linear differential equation to be exact (2 marks)
- ii. The condition for a nonlinear differential equation to be exact (2 marks)

b. By reducing the order , solve the nonlinear differential equation (16 marks)

$$x^2 y''' + (2x+xy)\, y'' + x(y')^2 + 3yy' = 0$$

QUESTION THREE (20MARKS)

a. i. Show that the functions $f_1(x) = -6x+2, f_2(x) = 6x^2-6x+1, f_1(x) = (x-1)$ are mutually orthogonal (6 marks)

ii. Find the orthonormal set for the functions in 2 a (i) (4 marks)

b. Write the Eigenvalue Problems as Sturm Liouville Problems (4 marks)

$$x^2 y'' + xy' + \lambda y = 0 \; ; \; y(1)=0, y(2)=0$$
$$y'' + 3y' + (\lambda+2)\, y = 0 \; ; \; y(1)=0, y(1)=2$$

c. Given that the general solution of the Bessel's ordinary differential equation is

$$J_n(x) = \sum_{m-n}^{\infty} \frac{(-1)^m}{m!\,(m-n)!} \left(\frac{x}{2}\right)^{2m-n}$$

Show that $\frac{d}{dx}\left[x^v J_v(x)\right] = x^v J_{v-1}(x)$ (6 marks)

QUESTION FOUR (20MARKS)

a. Solve the Boundary Value Problem for $\lambda = >0$

$$\frac{d^2 y}{dx^2} + 4\frac{dy}{dx} + 4y + 9\lambda y = 0$$
$$y(0) = 0 \,,\; y(1) = 0$$

(6 marks)

b. Show that the differential equation is exact and hence solve it (9 marks)

$$\frac{d^3 y}{dx^3} + \left(x+1+\frac{3}{x}\right)\frac{d^2 y}{dx^2} + 2\left(2+\frac{1}{x}\right)\frac{dy}{dx} + \frac{2}{x^2}\, y = 0$$

c. Consider the set of functions $\{1, Cosx, Cos2x\cdots\}$ on the interval $[-\pi, \pi]$ .Given that the norm of 1 is and the norm of $Cos\; nx\; is\; \sqrt[n]{\pi}$. Find the orthonormal set of $\{1, Cosx, Cos2x\cdots\}$. (5 marks)
