

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

**FIRST YEAR EXAMINATION FOR THE AWARD OF DEGREE OF DOCTOR OF
PHILOSOPHY IN APPLIED MATHEMATICS**

MATH 933: RECENT TRENDS IN DYNAMICAL SYSTEMS

**STREAMS: PHD MATHS
HOURS**

TIME: 3**DAY/DATE: THURSDAY 07/10/2021****2.30 P.M – 5.30 P.M.****INSTRUCTIONS:**

- Answer any **THREE** questions.

QUESTION ONE (20 MARKS)

(a) Define the following terms

- | | | |
|-------|----------------------------|-----------|
| (i) | Autonomous system | (2 marks) |
| (ii) | Critical point of a system | (2 marks) |
| (iii) | Phase plane | (2 marks) |

(b) An autonomous system has two Eigenvalue λ_1 and λ_2 , discuss the critical points in terms of nature and stability for

- | | | |
|------|----------------------|-----------|
| (i) | A linear system | (7 marks) |
| (ii) | Almost linear system | (7 marks) |

QUESTION TWO (20 MARKS)

Consider the non-linear dynamical system

$$\begin{aligned}\dot{x} &= x(1 - x - 2y) \\ \dot{y} &= y(1 - 2x - y)\end{aligned}$$

- | | | |
|-----|----------------------------------------------------------|------------|
| (a) | Determine all the fixed points. | (5 marks) |
| (b) | Discuss the stability of the fixed points obtained in a. | (15 marks) |

QUESTION THREE (20 MARKS)

(a) Outline the process of linearization of a non-linear system. (3 marks)

(b) For the non-linear dynamical system

$$\begin{aligned}\dot{x} &= -(x-y)(1-x-y) \\ \dot{y} &= x(2+y)\end{aligned}$$

(i) Linearize the system at all its equilibrium points. (9 marks)

(ii) Classifying the equilibrium points. (8 marks)

QUESTION FOUR (20 MARKS)

(a) Briefly explain the Mathematical meaning of the term almost linear system. (6 marks)

(b) Consider the system of equations

$$\frac{dx}{dt} = x$$

$$\frac{dy}{dt} = -x + 2y$$

(i) Verify that (0,0) is a critical point of the system. (3 marks)

(ii) Show that the system is almost linear. (3 marks)

(iii) Determine the Jacobian and then classify. (8 marks)
