CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF DOCTOR OF PHILOSOPHY IN APPLIED MATHEMATICS

MATH 932: MATHEMATICAL EPIDEMIOLOGY

STREAMS: **TIME: 3 HOURS**

DAY/DATE: WEDNESDAY 07/04/2021 **2.30 P.M – 5.30 P.M**

INSTRUCTIONS:

Answer Any Three Questions

QUESTION ONE.

a. Given that $S+I=N,\ S(0)=S_0,\ I(0)=I_0$, Solve the SI model below for I

$$\frac{dS}{dt}=-\beta SI$$
$$\frac{dI}{dt}=\beta SI$$

b. A typhoid fever model is given as $\frac{dy}{dt}=-\beta y \text{ and } \frac{dx}{dt}=-\alpha xy$, where x and y respectively denote the proportion of susceptible and carriers in the population. Suppose that the carriers are identified and removed from the population at a rate β.

i. Determine the proportions of carriers at any time t, where y(0)=y. [3marks]

ii. Use the result of part I above to find the susceptible at time t where $x(0)=x_0$ [5marks]

iii. Find the proportion of the population that escapes the epidemic. [2marks]

QUESTION TWO

a. Given the competing species model as,

$$\frac{dx}{dt} = r_1 x - \alpha_1 xy$$
$$\frac{dy}{dt} = r_2 y - \alpha_2 xy$$

Where α_1 and α_2 are two constants.

(i). obtain the solution for x in the absence of the second specie when y = 0 [3marks]

(ii). Obtain the non trivial steady state point of the system (x^*, y^*) [3marks]

iii. Examine the stability of the steady state (x^*, y^*) using the perturbation technique 10mks

iv. solve the competing model system in (2a) above analytically [4marks].

QUESTION THREE

a. Derive the predator prey model and explain the assumptions from the model developed [8marks]

b. Solve the predator prey model developed in (a) above stating the implications of the solutions obtained. [6marks]

c. Obtain the equilibrium points of the system below.

$$\frac{dx}{dt} = x - y + xy$$
$$\frac{dy}{dt} = 3x - 2y - xy$$

x - is the prey and y - is the predator. Verify that (0,0) is a critical point of the system and discuss the type and stability of the critical point (0,0) [6marks]

QUESTION FOUR

a. Solve the epidemic model given as $\frac{dx}{dt} = -x(n+1-x), where\ x = n \quad at \quad T = 0$, $x(t)$ is the number of susceptible, $y(t)$ is the number of infectives. [5marks]

b. If the contact rate $\beta = 0.001$ and the number of susceptible (n) is 2000 initially in 4a above, determine

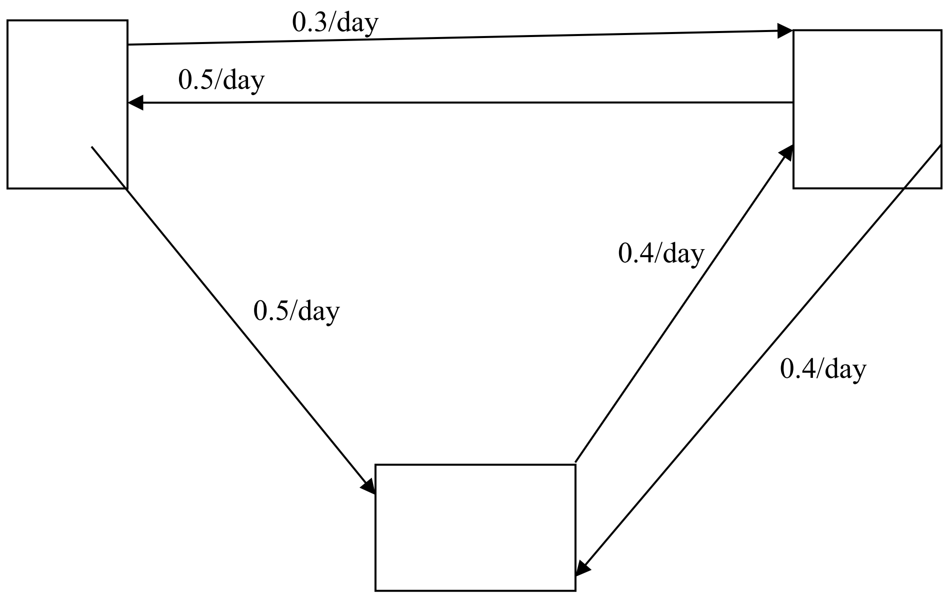
i. The number of susceptible left after 3 weeks [3marks]

ii. The density of susceptible when the rate of appearance of new cases is maximum [2marks]

iii. The time in weeks at which the rate of appearance of new cases is maximum [3marks]

iv. The maximum rate of appearance of new cases [2marks]

c. In the compartmental diagram given below, the transfers are made on a daily basis



Given that x(0) = [100, 0, 0], find the transfer matrix and state of the system over the next five days. [5marks]

QUESTION FIVE

a. Derive the logistic growth model for a single specific population. Given the differential equation $\frac{dN}{dt} = rN(1 - \frac{N}{K})$, where K is the carrying capacity, N(t) is the number of individuals in a population at time t and r is the growth rate, solve for N(t) [14marks]

b. Derive the Malthusian growth model and give its limitations [6marks]

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