CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF DOCTOR OF PHILOSOPHY IN APPLIED MATHEMATICS

MATH 929: DISTSRIBUTIONS AND FOURIER ANALYSIS

STREAMS: PhD **TIME: 3 HOURS**

DAY/DATE: WEDNESDAY 6/10/2021 **8.30 A.M – 11.30 A.M**

INSTRUCTIONS

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- **Answer any three questions**
- **Adhere to the instructions on the answer booklet.**

QUESTION ONE.

a. Solve the differential equation [7marks]

$$\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = x(t) \text{ given}$$

the input signal $x(t) = e^{-t}u(t)$ and the initial conditions $y(0) = 1$, $\dot{y}(0) = 1$:

b. Solve the differential equation below by finite Fourier transforms [13 marks]

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \text{ given } u(0, t) = 0 \text{ and } u(4, t) = 0$$

and $u(x, 0) = 2x$ where $0 < x < 4$, $t > 0$.

QUESTION TWO

a. Obtain the general solution to the following system of differential equations by the Mikusinski`s operator [10 marks]

$$x'(t) + y(t) = t^2 + 6t + 1$$

$$y'(t) - x(t) = -3t^2 + 3t + 1$$

b. Given the ordinary differential equation

$$-u''(x) + q(x)u(x) = \lambda u(x) \qquad \lambda \in \mathbb{C}\setminus\mathbb{R}$$

$$u_1(0,\lambda) = 0 \qquad \text{and} \qquad u_2(0,\lambda) = 1$$
$$u_1'(0,\lambda) = 1 \qquad\qquad\quad u_2'(0,\lambda) = 0$$

With q = 0, Obtain

i. The Titchmarsh-Weyl *m*-function [5marks]

ii. Spectral transform and measure [5 marks]

QUESTION THREE

a. Solve the differential equation [7marks]

$$\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = x(t)$$

Given the input signal

$x(t) = e^t u(t)$ and the initial conditions $y(0) = 1$, $\dot{y}(0) = 1$

b. Solve the differential equation below by Mikusinski`s operator [7marks]

$$y'(t) + 2y(t) = 2(t+1)e^{t^2}$$

with $y(0) = 1$.

c. Obtain the solution to the integral equation [6 marks]

$$f(t) = \cos t - t - 2 - \int_0^t (t-\tau)f(\tau)d\tau.$$

QUESTION FOUR

a. Solve the heat equation below by Fourier sine transforms [15 marks]

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

under the conditions

(i) $u(0, t) = 0$ *(ii)* $u(x, 0) = e^{-x}$
(iii) $u(x, t)$ *is bounded.*

b. Solve the differential equation below [5 marks]

$$\frac{\mathrm{d}}{\mathrm{d}t} f(t) + 2f(t) = 4(1 + a^2 t)\, \mathrm{e}^{a^2 t^2}$$

given that $f(0) = 3$.

$$\frac{\mathrm{d}}{\mathrm{d}t} f(t) + 2f(t) = 4(1 + a^2 t)\, \mathrm{e}^{a^2 t^2}$$

given that $f(0) = 3$.

QUESTION FIVE

a. Solve the transmission line equation below by Laplace transforms [20 marks]

$$\frac{\partial v}{\partial x} = -Ri$$

$$\frac{\partial i}{\partial x} = -c\frac{\partial v}{\partial t}$$

Conditions are $v\,(x, 0) = 0$

$i\,(x, 0) = 0$

$v\,(0, t) = v_0$

$v\,(x, t)$ finite for all x and t.
